

# **Managing portfolio managers: the impacts of market concentration, cross-sectional return dispersion and restrictions on short sales**

by  
Heidi Raubenheimer

*Dissertation presented for the degree of Doctor of Philosophy in  
Business Administration at the  
University of Stellenbosch*



Promoter: Prof Eon Smit  
University of Stellenbosch Business School

O ctej "4234

## **Declaration**

By submitting this thesis/dissertation, I declare that the entirety of the work contained therein is my own, original work, that I am the sole author thereof (save to the extent explicitly otherwise stated), that reproduction and publication thereof by Stellenbosch University will not infringe any third party rights and that I have not previously in its entirety or in part submitted it for obtaining any qualification.

Heidi Raubenheimer

March 2012

## **Abstract**

The impacts on the active management of investment portfolios of a) market concentration, b) cross-sectional return dispersion and c) restrictions on short sales are explored in this thesis. The context is the fund sponsor's management of their investment managers in a South African equity investment environment. Some of the findings here are developed analytically and some make use of multiple simulated investment views and their corresponding optimal portfolio solutions to document the size and nature of the inefficiencies that are created by these three factors.

The cross-sectional volatility of asset returns in an investment universe represents a carrying capacity for active risk taking: the higher the cross-sectional volatility, the greater the opportunity for active risk taking. Cross-sectional volatility is shown to be an important consideration when setting active risk targets. It is shown that, to remain efficient, active risk should be reduced during periods of low cross-sectional dispersion and vice versa. The sensitivity of active risk estimates to changes in the cross-sectional dispersion of their investment universe is demonstrated and sponsors should therefore exercise caution when reacting to changes in the active risk estimates of their funds. Cross-sectional volatility is shown to be time-varying and is related to similarly varying dispersion in realised fund returns. The ex post performance of competing portfolio managers therefore require correction for this heteroscedasticity and an effective weighted adjustment is recommended.

Active managers can only fully express their views in an environment where their mandated conditions accommodate their conviction and level of risk taking. The short sale restriction is shown to be materially binding when applied to a concentrated

benchmark such as the ALSI where only a few of the stocks comprise most of the total investment weight. The more concentrated the benchmark and the higher the active risk target, the wider the distribution of individual asset weights in the portfolio will be and the more binding the weighting constraints will be. It is shown that constraints on short positions are more binding on assets with small weightings in the benchmark illustrating the asymmetrical sub-optimal effect of these constraints when they are applied uniformly across the investment opportunity set. It is argued that requiring long-only managers to increase their active positions and/or active risk in a concentrated investment environment further constrains them in their ability to express their best investment view and increases their competitive disadvantage relative to unconstrained funds taking similar risk.

The research presented in this thesis measures the nature and size of the impacts of the market concentration, cross sectional return dispersion and restrictions on short sales that are implied by the investment mandate on the quality of the investment portfolio, providing analysis and techniques which can inform and improve the quality of the relationship between fund sponsor and fund manager. The more appropriate the investment mandate and the monitoring of the fund's performance subject to this mandate, the more effective the manager's risk-taking on behalf of their investors will be. This is the principle that this research aims to serve.

## Acknowledgements

The research questions in this dissertation (and the ones that will likely follow this work) were inspired by my participation in the investment industry in South Africa. The challenges in portfolio construction and managing client expectations experienced by the Smartcore team at Sanlam Investment Management (SIM) led to my exploration of these themes. I gratefully acknowledge the fierce intelligence, curiosity, energy and support of the Smartcore team at SIM in the production of this work.

I have had the privilege to present most of the work in this dissertation in some form, often multiple times and the dissertation has benefitted from the response of those who received and reviewed these presentations. In particular:

- the PhD colloquia at Stellenbosch Business School;
- European Doctoral Programmes Association in Management and Business Administration;
- the research seminars at the Graduate School of Business, the Department of Statistical Sciences and the Department of Actuarial Studies all at the University of Cape Town;
- the Actuarial Society of South Africa's Retirement Matters group;
- The Southern African Finance Association;
- the "Collective Insights" publication in FinWeek;
- Risk Magazine

Some of the work in this dissertation has been published:

- Raubenheimer, H. 2011. Constraints on Investment Weights: What Mandate Authors in Concentrated Equity Markets Such as South Africa Need to Know. *Investment Analysts Journal*, **74** (forthcoming).
- Raubenheimer, H. 2011. Varying Cross-Sectional Volatility in the South African Equity Market and the Implications for the Management of Fund Managers. *South African Journal of Business Management*, **42**(2), 15-26.
- Raubenheimer, H. 2010. Concentration in the South African Equity Market and its Implied Restrictions on the Long Only Equity Fund Managers Opportunity Set. *South African Journal of Business Management*, **41**(4).
- Raubenheimer, H. and Conradie, H.S. 2009. Be careful what you wish for – Active Risk is market dependent. Collective Insights, *FinWeek*, Autumn:17-19.

The contribution of the reviewers in the publication process is also acknowledged.

## Table of contents

<b>Abstract</b>	<b>iii</b>
<b>Acknowledgements</b>	<b>v</b>
<b>Table of contents</b>	<b>vii</b>
<b>List of tables</b>	<b>xi</b>
<b>List of figures</b>	<b>xii</b>
<b>List of equations</b>	<b>xvi</b>
<b>List of acronyms and abbreviations</b>	<b>xviii</b>
<b>Chapter 1 : Introduction</b>	<b>1-1</b>
1.1 OVERVIEW OR BACKGROUND	1-1
1.2. RESEARCH FOCUS	1-5
1.2.1 Research problem	1-9
1.2.1.1 Constrained domestic investment	1-11
1.2.1.2 Concentrated investment universe	1-12
1.2.1.3 Limited or no allowance for short selling	1-13
1.2.1.4 Varying dispersion in investment opportunities	1-14
1.2.2 Research questions	1-14
1.2.2.1 What are the inefficiencies implied by market concentration and long-only investment in the South African equity management?	1-14
1.2.2.2 Do markets and/or mandates have a carrying capacity for efficient active risk taking?	1-15
1.2.2.3 How can fund sponsors more fairly and accurately evaluate their managers' performance in the light of the changing cross-sectional dispersion of realised returns?	1-16
1.2.2.4 If fund sponsors were to consider relaxing the restriction on short sales, how much relaxation would be required and how would this improve the fund manager's ability to act on their investment forecasts?	1-16

1.2.3	RESEARCH AIM	1-17
<b>Chapter 2 : Literature review</b>		<b>2-18</b>
2.1	ACTIVE MANAGEMENT	2-18
2.1.1	The Fundamental Law of Active Management	2-23
2.1.2	The transfer coefficient	2-25
2.2	THE LONG-ONLY CONSTRAINT	2-28
2.3	CONCENTRATION	2-31
2.4	CROSS-SECTIONAL VARIANCE OF RETURNS	2-33
2.4.1	Importance to ex ante active management	2-35
2.4.2	Importance to ex post performance assessment	2-45
2.4.2.1	Risk-adjusted performance measurement	2-46
2.4.2.2.	Benchmark-adjusted performance measurement	2-48
2.4.2.3	Beyond benchmark adjustments	2-50
2.4.2.4	Heteroscedasticity in fund performance	2-51
<b>Chapter 3 Concentration in the South African equity market and its implied restrictions on the long-only equity fund manager's opportunity set</b>		<b>3-56</b>
3.1	SUMMARY	3-56
3.2	CONCENTRATION	3-56
3.2.1	Concentration in South Africa's equity market: present and historic	3-56
3.2.2	Concentration in South Africa's equity market relative to developed markets	3-64
3.3	THE IMPLIED RESTRICTIONS OF MARKET CONCENTRATION ON THE ACTIVE FUND MANAGEMENT OPPORTUNITY SET	3-68
3.3.1	The asymmetry of the long-only active portfolio manager's opportunity set	3-68
3.3.2	The effect of concentration in the presence of active weight limits	3-69
3.3.3	The implied transfer coefficient	3-76
3.3.4	Short extension funds	3-79
3.3.5	A comparison of South African equity indices	3-81
3.4	CONCLUSIONS	3-85
<b>Chapter 4 : Varying cross-sectional volatility in the South African equity market and the implications for the management of fund managers.</b>		<b>4-87</b>
4.1	SUMMARY	4-87
4.2	INTRODUCTION	4-87
4.3	HISTORICAL CROSS-SECTIONAL DISPERSION ON THE ALSI	4-88
4.3.1	Sectoral evidence	4-91



4.4	IMPLICATIONS OF CROSS-SECTIONAL VOLATILITY AND ITS CHANGES FOR PORTFOLIO MANAGEMENT AND MANDATE SETTING	4-93
4.4.1	Implications for mandate setting	4-94
4.4.2	Mandate compliance and forecast errors	4-100
4.5	CONCLUSIONS	4-104
<b>Chapter 5 : Varying cross-sectional volatility in asset returns and the implications for ex post performance measurement</b>		<b>5-106</b>
5.1	SUMMARY	5-106
5.2	INTRODUCTION	5-106
5.3	ANALYSIS	5-107
5.3.1	Data and sampling	5-107
5.3.2	Fund performance dispersion vs. benchmark dispersion	5-108
5.3.3	The influence of benchmark dispersion on OLS regression metrics	5-113
5.3.3.1	Test for heteroscedasticity	5-114
5.4	CORRECTING PERFORMANCE MEASUREMENT FOR HETEROSCEDASTICITY	5-116
5.4.1	Weighted regression	5-116
5.4.1.1	Performance rankings using regression metrics	5-118
5.4.2	Weighted Sharpe Ratios	5-120
5.4.2.1	Weighting simple multi-period returns	5-123
5.5	CONCLUSIONS	5-125
<b>Chapter 6 : Constraints on investment weights: What fund sponsors in concentrated equity markets such as South Africa need to know</b>		<b>6-129</b>
6.1	SUMMARY	6-129
6.2	INTRODUCTION	6-129
6.3	ACTIVE WEIGHTS AND THE RISK BUDGET	6-130
6.3.1	Optimal distributions of security weights	6-131
6.3.1.1	Unconstrained optimal portfolio weights	6-131
6.3.1.2	Optimal short weights	6-132
6.3.1.3	Distribution of optimal unconstrained active portfolio weights	6-133
6.3.3	Distribution of active weights under simplified conditions	6-134
6.3.3.1	Empirical demonstration using the ALSI and ALSI 40	6-136

6.3.4	Simulated active weight distributions	6-140
6.3.5	Fund level constraints on gearing and the risk budget	6-143
6.3.5.1	Two parameter covariance matrix simplification	6-143
6.4	CONCLUSIONS	6-147
<b>Chapter 7 : Simulating the impacts of cross-sectional return dispersion and the long-only constraint on funds that are benchmarked against the ALSI</b>		<b>7-149</b>
7.1	INTRODUCTION	7-149
7.2	DATA AND METHODOLOGY	7-151
7.2.1	Standard portfolio optimisation problem	7-151
7.2.1.1	The unconstrained optimal investment portfolio	7-151
7.2.1.2	Constrained optimisation	7-152
7.2.2	Data	7-154
7.2.2.1	Benchmark and investment universe	7-154
7.2.2.2	Alpha generation and the covariance matrix	7-155
7.2.3	Bases of Comparison	7-158
7.3	ANALYSIS	7-159
7.3.1	Transfer coefficient	7-159
7.3.2	Optimal gearing	7-164
7.3.3	Active share	7-169
7.3.4	Concentration	7-174
7.3.5	Weight not taken	7-178
7.4	CONCLUSIONS	7-181
<b>Chapter 8 : Conclusions and Recommendations</b>		<b>183</b>
<b>References</b>		<b>188</b>

## List of tables

Table 1.1: Regulatory global limits to foreign investment for a selection of countries .....	1-12
Table 3.1: Index weight of stocks on the ALSI and four other developed market indices ...	3-67
Table 3.2: Restricted opportunities at various active weight limits for the long-only investor in the ALSI .....	3-75
Table 3.3: Implied transfer coefficient of long-only funds with various FTSE/JSE benchmarks .....	3-84
Table 5.1: Ranked “alphas” using un-weighted and weighted regressions (5 year monthly return history to January 2011) .....	5-119
Table 5.2: Ranked Sharpe Ratios using un-weighted and weighted time series (5 year monthly return history to January 2011).....	5-122
Table 5.3: Breusch-Pagan regression results before and after adjusting for benchmark dispersion .....	5-127
Table 6.1: Sum of expected short positions (%) in an unconstrained optimal active ALSI fund.....	6-146
Table 6.2: Sum of expected short positions (%) in an unconstrained optimal active ALSI 40 fund.....	6-146

## List of figures

Figure 3.1: Market capitalisations of African stock exchanges.....	3-57
Figure 3.2: The distribution of (market capitalisation) weights on the ALSI (July 2011) ..	3-58
Figure 3.3 The cumulative distribution of (market capitalisation) weights on the ALSI (July 2011).....	3-58
Figure 3.4 The distribution of (market capitalisation) weights on the SWIX (July 2011) .	3-62
Figure 3.5 The cumulative distribution of (market capitalisation) weights on the SWIX (July 2011).....	3-62
Figure 3.6: The annual Rand value traded on the JSE .....	3-63
Figure 3.7: Concentration of the ALSI over time .....	3-63
Figure 3.8: Number and effective number of stocks on the ALSI over time .....	3-64
Figure 3.9: The Distribution of index weights on four major indices and the ALSI .....	3-65
Figure 3.10: Maximum number of stocks available at each level of active weight .....	3-72
Figure 3.11: Decreasing implied transfer coefficients with increasing active weight limits ..	3-76
Figure 3.12: Implied Transfer Coefficient for an ALSI equity fund.....	3-80
Figure 3.13: Implied Transfer Coefficient for an ALSI 40 equity fund.....	3-82
Figure 4.1: Historical cross-sectional standard deviation on the ALSI .....	4-90
Figure 4.2: Effective and actual number of stocks on the ALSI over time .....	4-90
Figure 4.3: Cross-sectional standard deviation on the FTSE/JSE sectors over time .....	4-92
Figure 4.4: Ratio of realised cross-sectional standard deviation to its one month lag...4-	102
Figure 4.5: Realised cross-sectional standard deviation relative to increasing lags .....	4-103
Figure 5.1: Fund performance dispersion around the benchmark .....	5-109
Figure 5.2: Cross-sectional dispersion on the ALSI and dispersion in the active returns of General Equity mutual funds.....	5-111

Figure 5.3: Scatter plot of the cross-sectional standard deviation of fund performance and the weighted cross-sectional standard deviation of the benchmark assets.....	5-112
Figure 5.4: Plot of the distribution of OLS residuals across all General Equity mutual funds .....	5-114
Figure 5.5: Weightings to adjust the annual performances of General Equity funds over various investment horizons .....	5-125
Figure 6.1: 95% Confidence intervals for optimal unconstrained holdings in each benchmark security ( $\sigma\sqrt{1-\rho} = 0.35$ , active risk=4% p.a., benchmark: ALSI) .....	6-138
Figure 6.2: 95% Confidence intervals for optimal unconstrained holdings in each benchmark security ( $\sigma\sqrt{1-\rho} = 0.75$ , active risk=4% p.a., benchmark: ALSI) .....	6-138
Figure 6.3: 95% Confidence intervals for optimal unconstrained holdings in each benchmark security ( $\sigma\sqrt{1-\rho} = 0.35$ , active risk=4% p.a., benchmark: ALSI 40) .....	6-139
Figure 6.4: 95% Confidence intervals for optimal unconstrained holdings in each benchmark security ( $\sigma\sqrt{1-\rho} = 0.75$ , active risk=4% p.a., benchmark: ALSI 40) .....	6-139
Figure 6.5: Range of simulated optimal unconstrained holdings in each benchmark security: covariance matrix: July 2008, 4% target active risk. ....	6-142
Figure 6.6: Range of simulated optimal unconstrained holdings in each benchmark security: covariance matrix: July 2011, 4% target active risk. ....	6-142
Figure 7.1: Distribution of TC across all simulated constrained portfolios as a function of active risk-high cross-sectional standard deviation: $\sigma\sqrt{1-\rho} = 0.50 p.a.$ .....	7-161
Figure 7.2: Distribution of TC across all simulated constrained portfolios as a function of active risk- low cross-sectional standard deviation: $\sigma\sqrt{1-\rho} = 0.25 p.a.$ .....	7-161
Figure 7.3: Distribution of TC across all simulated constrained portfolios as a function of active risk (covariance matrix: July 2008) .....	7-162

Figure 7.4: Distribution of TC across all simulated constrained portfolios as a function of active risk (covariance matrix: July 2011) .....	7-162
Figure 7.5: Distribution of optimal gearing across all simulated unconstrained portfolios as a function of active risk - high cross-sectional standard deviation: $\sigma\sqrt{1-\rho} = 0.50 p.a.$ ...	7-166
Figure 7.6: Distribution of optimal gearing across all simulated unconstrained portfolios as a function of active risk- low cross-sectional standard deviation: $\sigma\sqrt{1-\rho} = 0.25 p.a.$ .....	7-166
Figure 7.7: Distribution of optimal gearing across all simulated unconstrained portfolios as a function of active risk (covariance matrix: July 2008).....	7-167
Figure 7.8: Distribution of optimal gearing across all simulated unconstrained portfolios as a function of active risk (covariance matrix: July 2011).....	7-167
Figure 7.9: Distribution of active share across all simulated constrained portfolios as a function of active risk - high cross-sectional standard deviation: $\sigma\sqrt{1-\rho} = 0.50 p.a.$ .....	7-172
Figure 7.10: Distribution of active share across all simulated constrained portfolios as a function of active risk- low cross-sectional standard deviation: $\sigma\sqrt{1-\rho} = 0.25 p.a.$ .....	7-172
Figure 7.11: Distribution of active share across all simulated constrained portfolios as a function of active risk (covariance matrix: July 2008).....	7-173
Figure 7.12: Distribution of active share across all simulated constrained portfolios as a function of active risk (covariance matrix: July 2011).....	7-173
Figure 7.13: Distribution of concentrations of all simulated constrained portfolios as a function of active risk (high cross-sectional standard deviation: $\sigma\sqrt{1-\rho} = 0.50 p.a.$ .....	7-176
Figure 7.14: Distribution of concentrations of all simulated constrained portfolios as a function of active risk - low cross-sectional standard deviation: $\sigma\sqrt{1-\rho} = 0.25 p.a.$ .....	7-176
Figure 7.15: Distribution of concentrations of all simulated constrained portfolios as a function of active risk (covariance matrix: July 2008).....	7-177

Figure 7.16: Distribution of concentrations of all simulated constrained portfolios as a function of active risk (covariance matrix: July 2011).....	7-177
Figure 7.17: Average weights not taken of all simulated constrained portfolios (covariance matrix: July 2008) .....	7-180
Figure 7.18: Average weights not taken of all simulated constrained portfolios (covariance matrix: July 2011) .....	7-180

## List of equations

Equation 2.1: Definition of realised active return.....	2-18
Equation 2.2: Definition of active weight .....	2-18
Equation 2.3: Definition of portfolio risk.....	2-19
Equation 2.4: Definition of active risk .....	2-20
Equation 2.5: Capital Asset Pricing Model (CAPM) .....	2-20
Equation 2.6: Active management optimisation problem .....	2-21
Equation 2.7: Unconstrained optimal active weights for a given target active risk .....	2-22
Equation 2.8: Alpha generation (Grinold, 1994).....	2-24
Equation 2.9: Fundamental Law of Active Management (Grinold, 1994) .....	2-24
Equation 2.10: Definition of Transfer Coefficient (TC) .....	2-26
Equation 2.11: The Generalised Fundamental Law of Active Management .....	2-26
Equation 2.12: TC under full covariance assumptions .....	2-27
Equation 2.13: Effective number of stocks in an index/benchmark.....	2-31
Equation 2.14: Relationship between market dispersion and security dispersion.....	2-40
Equation 2.15: Relationship between portfolio dispersion and security dispersion .....	2-41
Equation 2.16: The relationship between cross-sectional volatility, average volatility and average correlation .....	2-43
Equation 2.17: The relationship between unexpected change in cross-sectional volatility and realised active risk .....	2-44
Equation 2.18: Sharpe ratio (Sharpe 1966) .....	2-46
Equation 2.19: t-statistic testing the hypothesis of superior risky performance .....	2-47
Equation 2.20: Information Ratio .....	2-49
Equation 2.21: paired t-statistic testing the hypothesis of zero performance difference ..	2-50
Equation 3.1: Definition of Implied Transfer Coefficient (ITC) .....	3-78
Equation 3.2: Generalised ITC allowing for short extension (X).....	3-79



Equation 4.1: Realised cross sectional variance .....	4-88
Equation 4.2: Active risk components of an unconstrained portfolio .....	4-95
Equation 4.3: Ex ante active risk estimation .....	4-101
Equation 5.1: OLS performance analysis.....	5-113
Equation 5.2: Breusch-Pagan regression to test for heteroscedasticity related to benchmark dispersion.....	5-115
Equation 5.3: Weighted OLS regression.....	5-117
Equation 5.4: Weighted Returns .....	5-124
Equation 6.1: Condition for a short position in an unconstrained optimal fund.....	6-132
Equation 6.2: Alpha generation a la Clarke <i>et al.</i> (2006).....	6-133
Equation 6.3: Optimal unconstrained active weight distribution under the two-parameter covariance assumption .....	6-135
Equation 6.4: Expected short selling weight conditional on short sales being required (Clarke <i>et al.</i> , 2008) .....	6-144
Equation 7.1: Typical portfolio equality constraints .....	7-152
Equation 7.2: The long only portfolio constraint .....	7-154
Equation 7.3: Gearing .....	7-165
Equation 7.4: Active share .....	7-170
Equation 7.5: Effective number of stocks.....	7-175
Equation 7.6: Weight not taken.....	7-179

## List of acronyms and abbreviations

AGL	Anglo American PLC
ALSI	FTSE/JSE All-Share Index
ALSI 40	FTSE/JSE Top 40 Index
bp	basis points i.e. decimal points of a percentage. E.g. 10bp = 0.10%.
BIL	BHP Billiton PLC
CAPI	FTSE/JSE Capped Index
CAPM	Capital Asset Pricing Model
CEO	Chief Executive Officer
FINDI	FTSE/JSE Financial and Industrial Index
FINI	FTSE/JSE Financial Index
GIPS	Global Investment Performance Standards (GIPS®).
HHI	Herfindahl–Hirschman Index
IC	Information Coefficient
INDI	FTSE/JSE Industrial Index
IR	Information Ratio
ITC	Implied Transfer Coefficient
JSE	Johannesburg Stock Exchange
MPT	Modern Portfolio Theory
OLS	Ordinary Least Squares
OECD	Organisation for Economic Co-Operation and Development
PODS	Portfolio Opportunity Distributions
RESI	FTSE/JSE Resources Index
SWIX	FTSE/JSE Shareholder Weighted Index
TC	Transfer Coefficient

## Chapter 1 : Introduction

### 1.1 OVERVIEW OR BACKGROUND

“Do what you will, the capital is at hazard...All that can be required of a trustee to invest, is, that he shall conduct himself faithfully and exercise a sound discretion. He is to observe how men of prudence, discretion, and intelligence manage their own affairs, not in regard to speculation, but in regard to the permanent disposition of their funds, considering the probable income, as well as the probable safety of the capital to be invested.” Justice Samuel Putnam (1830)<sup>1</sup>.

This quote by Justice Putnam has come to be immortalised as the “Prudent Man Rule” and applies to those who administer funds on behalf of the beneficiaries or owners of those funds. Putnam issued this judgement in 1830 in response to a lawsuit lodged by the beneficiaries of the estate of a wealthy Bostonian concerning the estate’s administration. This Prudent Man Rule epitomises both the sense of the responsibilities of investment professionals and the concept of investment risk before the influence of Markowitz, the 1990 Nobel Prize winner in Economic Sciences. Prior to Markowitz (1952), investment activity was understood to be risky and, although risk was not necessarily measured numerically, “prudent” trustees who invested on behalf of others were required to be mindful of it. However, the failure of the beneficiaries to successfully sue the administrators of the fund in question in this 1830 judgement, despite the fund’s 40% loss of value, recognised that if trustees could be held liable for losses that were not “owing to their wilful default”, there would be few people willing to take such responsibility. The role

---

<sup>1</sup> (Bernstein, 1996: 248)

and responsibility of those who manage the funds of others prudently was thus demarcated.

Markowitz's innovation in the arena of fund management some 120 years later was his consideration of the entire portfolio of investments rather than the more typical method at the time of managing each investment as an individual entity. Standing on the shoulders of giants such as Bayes, Gauss, Bernoulli, Von Neumann and Morgenstern, this economist developed what later became known as Modern Portfolio Theory (MPT).

"My 1952 article on portfolio selection proposed expected (mean) return... and variance of return... of the portfolio as a whole as criteria for portfolio selection, both as a possible hypothesis about actual behaviour and as a maxim for how investors ought to act. The article assumed that "beliefs" or projections about securities follow the same probability rules that random variables obey." (Markowitz, 1999: 5)

Modern Portfolio Theory presented fund managers with a decision framework based on a multivariate normal model of investment returns. The investment decision-maker could construct an investment portfolio consistent with their forecasts and uncertainties of the investments in their universe using Markowitz' mean variance paradigm for portfolio selection. Furthermore, Markowitz showed that the covariances of investment returns are material to the selection and weighting of investments within a portfolio and provide opportunities for reducing risk to optimal and efficient levels.

Tobin (1958) and Sharpe (1964) extended this framework by articulating the decision maker's trade-off between riskless and risky assets. One of the features of Tobin's Separation Theorem and Sharpe's Capital Asset Pricing Model (CAPM) is the comparison of a fund manager's choices to an investment in a riskless asset (in practice, a cash investment). Both used the riskless asset (and its risk-free rate of return) as a tool for calibrating the amount of risk that a rational, profit maximising investment decision-maker should procure depending on their particular appetite for risk-taking.

As the practice of portfolio management developed, so did the number of portfolio managers and the competition for assets for these managers to manage. With a competing field for this professional service, came the need to restate the decision-making paradigm to one that better represented the decisions of competing professional managers in the role of investment agent. The riskless asset may remain a suitable comparison for the investor's decision but for professional fund managers the "riskless" position is the benchmark portfolio. The benchmark portfolio represents the neutral investment position that satisfies the client's instructions without taking any particular investment view. The benchmark portfolio is "passive" and represents an absence of business risk on the part of the "active" portfolio manager: if you invest only in the benchmark portfolio, there is no risk of acting on an incorrect return forecast. This new active management framework was developed by Grinold and Kahn whose book (Grinold and Kahn, 2000) based on their earlier publications (Grinold, 1989), (Grinold, 1994) and (Goetzmann, Grinold and Kahn, 1996)) became the bible of professional fund managers.

"The modern portfolio theory taught in most MBA programs looks at total risk and total return. The institutional investor in the United States and to an increasing extent

worldwide cares about active risk and active return...The focus on active management arises for several reasons:

- Clients can clump the large number of investment advisers into recognizable categories. With the advisers thus pigeonholed, the client (or consultant) can restrict searches and peer comparisons to pigeons in the same hole.
- The benchmark acts as a set of instructions from the fund sponsor, as principal, to the investment manager, as agent. The benchmark defines the manager's investment neighbourhood. Moves away from the benchmark carry substantial investment and business risk.
- Benchmarks allow the trustee or sponsor to manage the aggregate portfolio without complete knowledge of the holdings of each manager. The sponsor can manage a mix of benchmarks, keeping the "big picture". (Grinold and Kahn, 2000: 4 - 5).

Successful active management relies on investing in superior forecasts of returns that depart from equilibrium. The ability to profit from the opportunities available in less than perfectly efficient markets is the way active managers distinguish themselves from their competition. It is up to the investor to manage their total risk exposure and to select and instruct each of their active managers appropriately. Active managers busy themselves with the trade-off between active (excess-of-benchmark) return and active risk (the variability of the fund returns' difference from the benchmark).

"Quantitative active management is the poor relation of modern portfolio theory. It has the power and structure of modern portfolio theory without the legitimacy. Modern portfolio theory brought economics, quantitative methods, and the scientific perspective to the study

of investments. Economics, with its powerful emphasis on equilibrium and efficiency, has little to say about successful active management.” (Grinold and Kahn, 2000: 3 - 4).

The work of this thesis falls squarely in the domain of active management, accepting its flaws and acknowledging its centrality to the business of managing the funds of others as it is practised today. The focus of this thesis is to inform the relationship between the fund manager and their client within the active management framework that is typical of modern fund management.

## 1.2. RESEARCH FOCUS

The relationship between the modern professional fund manager and fund sponsor is a classic **principal-agent** relationship. The fund sponsor or the fund’s trustees are the “owners” of the funds: they either actually own the assets that are being managed or, in the case of a pension or endowment, they represent the fund’s beneficiaries or members. The fund sponsor therefore acts as (or is) the beneficiary of the fund’s profits or losses. The fund manager is effectively an agent of the fund who earns a professional fee (and sometimes a performance-related fee) for their service to the fund and who bears the risk of dismissal. The fund sponsor therefore takes the role of the shareholder (principal) and the fund manager that of the professional CEO (agent), employed to manage a firm that is not their own.

In keeping with the majority of literature in the area of fund management, the analysis in this thesis uses standard quantitative and operational research techniques: decision analysis, optimisation, and simulation amongst others. The purpose of this analysis is to

inform the decision-making relationship between principal and agent in the fund management context, hence the title of “Managing fund managers”.

Arguably, the activities of a good manager in any field should include:

- i) the setting of achievable and reasonable goals and **objectives**;
- ii) the setting of prudent limits and **constraints** and enabling good risk management;  
and
- iii) the fair and objective measurement of **performance**, retrospectively.

As a manager of fund managers, the fund sponsor participates in each of these management activities. The fund sponsor determines the overall objectives of the fund and expresses this by way of an investment policy statement. The sponsor will then engage one or many fund managers to manage an allocated portion of the fund in way that is consistent with the overall investment policy. The contractual agreement with each fund manager takes the form of an investment mandate. (The professionals who participate in the management of funds are employees of investment companies and would be subject to many different mandates for many different funds but for the purposes of this argument, the fund manager is an agent of the fund sponsor and the investment mandate describes their relationship with the sponsor).

The mandate therefore describes the decisions made by the sponsor with respect to each of the three management activities listed above. Mandates representing the same fund sponsor and the same overall fund would not necessarily be consistent with each other but would be collectively consistent with the overall needs of the fund as a whole. The fund sponsor would also ensure that all their mandate agreements collectively and individually



satisfied any regulatory constraints that apply to the overall fund. In particular, each mandate would describe the following aspects of the fund's management:

- i) the investment universe and benchmark portfolio;
- ii) the risk budget;
- iii) the investment constraints; and
- iv) the method of performance measurement.

The **investment universe** stipulates where the manager may invest and where they may not. For example, a fund manager may be required to invest only in domestic equities listed on the main board of the JSE. The fund sponsor in this case may have specially selected the manager for his/her skill in domestic equity management and the sponsor would have other managers invested in complementary asset classes and geographical locations. It is important then for the management of the fund as a whole that this portfolio manager remains invested in domestic equities, regardless of the investment risks and opportunities outside of his/her investment universe.

The **benchmark** portfolio is a notional investment portfolio that represents the investment universe and the neutral expectations of the fund sponsor: if the fund manager sees no opportunities for superior investment return, the fund sponsor expects that manager to invest in the benchmark portfolio and to produce the same return as the benchmark. The benchmark represents a passive or unmanaged alternative to the managed portfolio and, as such, the choice of benchmark is an important element of performance monitoring the manager's success. It is common practice to use a published market index or some combination of indices as a benchmark since they are investable, their composition is known a-priori and they are provided by a source independent to the manager. The

benchmark is integral to the monitoring of the activities of the fund manager ex ante and measuring the performance of the fund under management ex post. As such, it is important for the benchmark to be well defined in the mandate.

The sponsor will give an indication of the fund manager's **risk budget**: their expected level or range of active risk for the funds under management. On one hand, this budget limits the activities of the fund manager by avoiding more risk than the sponsor is willing to bear with those funds. On the other hand, it sets an objective for the level of activity that the sponsor requires of the fund manager: if the active risk is lower than the sponsor expects, it suggests that the fund manager is not sufficiently engaged in active investment opportunities (investing in mispricing and non-equilibrium views) relative to a passive benchmark investment. Fund sponsors expect active managers to be active.

In addition to the limits on the investment universe and risk, the fund sponsor will add specific **constraints** to the mandate. Sometimes these investment constraints are required by the regulations governing the type of fund in question, such as the restrictions described by Regulation 28 to the Pension Funds Act which governs South African pension fund assets. In these cases, the mandate will require compliance to the relevant regulations along with any additional fund-specific constraints. For example, the mandate might require an upper limit to the relative size of the investment in any particular asset or group of assets to ensure diversification of risk, prohibiting the fund manager from "putting all their eggs in one basket". Some mandates require sector-neutrality in which case the collective investment weights across the stocks of a sector are restricted to their weight in the market. In these cases, the sponsor is expressing the objective that the fund manager engage in stock selection but not sector speculation. A socially conscious sponsor, as

another example, may supply a list of companies or sectors in which they are unwilling to invest, regardless of the manager's ability to add value using these forbidden assets. Constraints therefore limit the activity of the skilled fund manager to accommodate the objectives, concerns and outcomes of the fund sponsor (and regulators).

The sponsor and fund manager will also agree up front as to the way in which the **performance** of the funds under their management will be monitored, measured and assessed. Typically, the sponsor will require certain performance reporting standards such as GIPS<sup>2</sup> compliance. They may also require certain reporting models to be used such as the MSCI Barra Aegis South African Equity Risk Model in order to facilitate their ability to compare performances among different managers fairly. Ideally, the method of performance measurement chosen will be objective, fair and consistent with the objectives and constraints of the fund.

### 1.2.1 Research problem

The management function of the fund manager is the allocation of the resources under their management (the value of the funds they are managing) in a way that serves the fund's objectives and complies with the fund's restrictions. As such, the successful fund manager invests in investment opportunities that will be profitable and avoids investments that will incur losses. Without overstepping the confines of their mandate, the fund manager invests and dis-invests varying proportions of the fund in a way that is most consistent with his/her forecasts of the future profitability of the cross-section of assets at their disposal.

---

<sup>2</sup> Global Investment Performance Standards (GIPS®).

The success (and failure) of fund managers' activities is obscured by the inherent randomness and volatility of the market in which they exercise their forecasting skill. It is difficult to distinguish between luck and skill on the part of a professional fund manager when evaluating their realised performance. Furthermore, and more particular to this research problem, the investment mandates themselves can diminish and distort the ability of fund managers to exercise their skill. This happens when the requirements of the fund sponsor as expressed in the mandate are inconsistent with each other and/or with the fund's objectives. When the investment constraints, the risk budget and the investment universe are ill suited to each other, the mandates can materially change the way in which fund managers are able to act on their investment forecasts. In their struggle to comply with their mandates, the fund managers may be forced to act inconsistently with their forecasts and the resulting performances of their funds will be an inaccurate reflection of their investment abilities.

This research **aims** to add to the body of research that supports the management of fund managers, specifically the setting of investment objectives and constraints, and the monitoring and measurement of investment performance.

The bulk of professionally managed investment assets in South Africa belong to pension funds and the bulk of these assets are invested in South African equities. The successful management of the pensionable wealth of an emerging country such as South Africa with a broadening base of beneficiaries of this management is of particular concern to our economy and to the professional investment community in South Africa as a whole. For this reason, the particular problem context in this thesis is general domestic equity fund management. The problems explored in this thesis however, are relevant in other

markets and other asset classes as well and the analyses presented throughout need only be replicated in each particular investment context to determine their extent.

In particular, this research concerns itself with the influence of the following features of South African pension fund management individually and in combination as they relate to the management of fund managers:

- i) the requirement to retain the bulk of South African assets within South Africa;
- ii) the concentration of the South African equity offering;
- iii) the restriction against short<sup>3</sup> investment positions; and
- iv) the varying cross-sectional dispersion of realised returns.

#### **1.2.1.1 Constrained domestic investment**

South African investors are restricted from unconstrained global diversification by exchange controls. South Africa's regulations governing pension fund investments were recently amended (2011) to allow for a maximum foreign investment (across all asset classes) of 25%. It is not uncommon for countries to restrict their pension funds' investments to domestic markets in this way. According to a 2010 Survey of Investment Regulations of Pension Funds<sup>4</sup>, the Russian Federation doubled their pension funds' foreign investment allowance from 10% to 20% in 2010; Brazil only allows for foreign exposure by way of retail funds and those are required to hold at least 80% in Brazilian

---

<sup>3</sup> A short position is the sale of an investment which is not already owned. The investor holding a short position is hoping to profit from a decrease in the investments value by selling it at its present price with the intention of buying it back later at a lower price. Essentially, a short position in an asset is a loan against the asset.

<sup>4</sup> 2010 Survey of Investment Regulations of Pension Funds, June 2011, Organisation for Economic Co-Operation and Development [online] <http://www.oecd.org/dataoecd/30/6/43939773.pdf>

foreign debt. Some of the other countries included in this OECD survey are listed in Table 1.1 along with their global foreign investment limits.

**Table 1.1: Regulatory global limits to foreign investment for a selection of countries**

Country	Upper Limit
Poland	5%
Mexico	20%
Russia	20%
Austria	30%
Korea	30%
Switzerland	30%
Colombia	40%
Chile	65%

Aside from regulatory constraints preventing geographical diversification, it is not unusual to write mandates specific to a geographical region while the fund as a whole is managed across globally diversified mandates. Therefore, the problem setting presented here is applicable to every fund management setting where mandates materially restrict the investment universe.

### **1.2.1.2 Concentrated investment universe**

A small number of large stocks comprise a substantial portion of the capitalisation of the South African equity offering. Active management concerns the allocation of relatively positive and relatively negative positions with respect to an equity benchmark. As such, an active manager is not particularly concerned with the relative size of companies. However, active investment in larger stocks is relatively easier to achieve than active positions in smaller stocks, more particularly when short positions are restricted or forbidden. Consequently, an asymmetry is introduced into what is theoretically a symmetrical decision framework.

Once again, concentration exists in any investment universe in which capitalisations are not uniformly distributed. Mandates that use anything other than equally weighted benchmarks will suffer from concentration-related issues in the same way that fund managers with South African equity mandates do but perhaps to a lesser extent. The concentration of the South African equity market is profound and the results presented in this thesis are therefore of particular importance to the management of fund managers who are confined to such a markets.

### ***1.2.1.3 Limited or no allowance for short selling***

In South Africa, as with many conservative investment domains, there is a regulatory restriction against incurring a liability on behalf of a pension fund i.e. the fund is not permitted to incur debt. In addition to this regulation, pension funds usually have as part of their own fund rules that no short selling is permitted within the fund. While these rules may well change in the near future to allow limited short selling, it is still almost always the case that fully invested funds in South Africa (both pension and retail funds) are restricted to long-only investment at an asset level.

This thesis is particularly concerned with the effect of this constraint on the management of fund managers. While long-only management is common practice in many fund management contexts and domiciles, when combined with constrained investment within a concentrated equity offering, the effect of this constraint is substantial and therefore of particular importance to the business of managing South African pension fund managers.

#### **1.2.1.4 *Varying dispersion in investment opportunities***

Professional fund managers rely on the dispersion of investment opportunities at any given time. Without differences among assets in forecasted and realised returns, fund managers would be unable to take a meaningful active investment position in any asset that differed from their position in any other asset and they would have no capacity for competing with other, similarly mandated managers.

Realised returns are indeed dispersed among assets but the size of this dispersion changes over time i.e. cross-sectional returns are heteroscedastic. This research is concerned with the variation in this cross-sectional dispersion and its varying effect on fund management. All things being equal, when dispersion is relatively low, the detrimental effects of concentration and short sale restrictions are greater than when dispersion is high. This heteroscedasticity also has a material influence on the realised performance of competing fund managers and accommodating this variation in dispersion is important in the fair assessment of ex post fund performance.

### **1.2.2 Research questions**

#### **1.2.2.1 *What are the inefficiencies implied by market concentration and long-only investment in the South African equity management?***

South Africa's equity market provides a large (in terms of volume) but concentrated investment environment. Domestic pension funds are restricted from diversifying globally and they are consequently faced with a restricted set of investment opportunities. This question (addressed primarily by the work in Chapter 3) quantifies the extent and effect of the concentration on the ALSI historically and at present and the impact on the long-only



fund manager's opportunity set. Chapter 6 explores the distribution of likely investment weights in a portfolio when no restrictions are placed on portfolio holdings in order to demonstrate the nature of the long-only restriction in the South African equity environment. Finally, Chapter 7 compares portfolios constructed with and without the long-only constraint in this same environment and quantifies the relative inefficiencies of long-only investments.

### ***1.2.2.2 Do markets and/or mandates have a carrying capacity for efficient active risk taking?***

This question borrows from the biological concept of “carrying capacity”: the capacity of an ecosystem to sustain the population of a species without adverse consequences for the environment. In a fund management context, this question asks whether there is a carrying capacity for efficient active risk-taking implied by a mandate and the investment universe to which it applies. This question (addressed primarily by the work in Chapter 4) particularly addresses the role of the varying cross-sectional dispersion in the investment universe and its influence on the risk budgeting of efficient fund managers. By implication, if the fund sponsor requires greater risk than the carrying capacity of the investment universe allows, there will be a detrimental impact on the management of the fund and this impact needs to be understood. Chapter 4 and Chapter 7 will demonstrate the increasing inefficiency of constrained portfolios with increasing risk budgets, particularly when the carrying capacity is low.

***1.2.2.3 How can fund sponsors more fairly and accurately evaluate their managers' performance in the light of the changing cross-sectional dispersion of realised returns?***

If the investment universe provides a varying opportunity set for fund managers, as suggested by the previous question (1.2.2.2), by implication the ex post performance analysis of these fund managers will be influenced by this variation. This question (addressed in Chapter 5) asks whether there is a way to more fairly and accurately assess the managers' success from their ex post performance in the light of the varying cross-sectional dispersion of realised returns.

***1.2.2.4 If fund sponsors were to consider relaxing the restriction on short sales, how much relaxation would be required and how would this improve the fund manager's ability to act on their investment forecasts?***

In order to answer this question, the nature of the distribution of investment weights must be explored. "Short extension" mandates allow portfolios to have a limited collection of short sales provided that the net portfolio is fully invested. The aim of this so-called "short extension" is to improve the efficiency of the implementation of the fund manager's investment view. This question seeks to explore the effectiveness of such a short extension in an investment environment as highly concentrated as the South African equity market. When is such a short extension likely to be required, with respect to which assets and how much shorting is likely to be needed?

Using available theory and simulation, the analysis (presented in Chapter 3 and Chapter 6) demonstrates a way to define prudent expectations and restrictions on individual shorting requirements per stock to inform the setting of mandates for short extension products in

the context of South African equity fund management. These expectations and restrictions are, in turn, dependent on the cross-sectional volatility of the investment universe as well as the concentration of the benchmark and active risk target.

### 1.2.3 RESEARCH AIM

By measuring the nature and size of the impact of market concentration, cross sectional return dispersion and restrictions on short sales on the quality of the investment portfolio, this research aims to inform and improve the quality of the relationship between fund manager and fund sponsor. The contribution of this thesis is ultimately to improve the mandate between fund sponsor and fund manager with respect to:

- i) the setting of prudent, appropriate and internally consistent investment **objectives and constraints**; and
- ii) the improved assessment of ex post investment **performance**.

The more appropriate the investment mandate and the monitoring of the fund manager's performance subject to this mandate, the more effective the risk-taking on behalf of the investors/pensioners will be. Therefore, while the broad aim of this research is the improvement of the quality of investment mandates, the natural beneficiary of this research is ultimately the investor.

## Chapter 2 : Literature review

### 2.1 ACTIVE MANAGEMENT

Goetzmann *et al.* (1996) defined the term “active management”. The concept of “active returns” replaced modern portfolio theory’s paradigm of returns. Active managers pursue, not returns in excess of the risk-free rate, but returns in excess of the benchmark i.e. active return.

#### Equation 2.1: Definition of realised active return

$$R_{a,i} = R_i - R_b$$

where  $R_i$  and  $R_b$  are the realised return of the asset ( $i$ ) and the benchmark respectively and  $R_{a,i}$  is the active return of asset ( $i$ ).

The MPT paradigm aims to find the investment weights that describe a portfolio that satisfies the investor’s risk and return requirements. In an active management framework, the portfolio selection problem is solved in terms of active weights i.e. the weights that describe the chosen portfolio’s departure from the benchmark portfolio.

#### Equation 2.2: Definition of active weight

$$w_{a,i} = w_{p,i} - w_{b,i}$$

where  $w_{p,i}$  is the relative size (weight) of the portfolio’s investment in asset,  $i$ ;

$w_{b,i}$  is the weight of the benchmark in the same asset; and

$w_{a,i}$  is the active weight of the portfolio in asset ( $i$ ).

In the MPT paradigm, rational investors select from risky and riskless investments. The risk of an investment portfolio is calculated using the weightings of the investments in risky assets and the covariance matrix of investment<sup>5</sup>. The weight of the investment in the risk-free asset simply increases/decreases the overall portfolio risk in a linear way.

### Equation 2.3: Definition of portfolio risk

$$\sigma_p = (\mathbf{w}_p' \boldsymbol{\Sigma} \mathbf{w}_p)^{\frac{1}{2}}$$

where  $\sigma_p$  is the risk/uncertainty of the investment portfolio's return;

$\mathbf{w}_p$  is an  $n \times 1$  vector of portfolio weights ( $w_{p,i}$ ) in  $n$  risky assets; and

$\boldsymbol{\Sigma}$  is an  $n \times n$  covariance matrix of the returns of these same assets

In the active management framework, managers select from the passive, benchmark holdings and a set of active positions (refer Chapter 1 Section 1.1). In this active management paradigm, the riskless position is no longer the risk-free asset,  $r_f$ , but rather the benchmark portfolio. The active risk of this passive benchmark investment ( $\mathbf{w}_b$ ) is zero and the active risk of any other portfolio ( $\mathbf{w}_p$ ) is the uncertainty of the difference between the portfolio's return and the benchmark's return.

---

<sup>5</sup> This formulation is accurate whether risk is measured ex post or estimated ex ante. Ex post measurement of portfolio risk would use the weightings at the beginning of the performance period and the covariance matrix of realised returns over the period. Ex ante estimation would make use of a forecast covariance matrix of returns and the weightings at the beginning of the forecast period.

**Equation 2.4: Definition of active risk**

$$\sigma_a = (\mathbf{w}_a' \Sigma \mathbf{w}_a)^{\frac{1}{2}}$$

where  $\sigma_a$  is the active risk of the investment portfolio and  $\mathbf{w}_a$  is an  $n \times 1$  vector of active weights ( $w_{a,i}$  as per Equation 2.2).

Sharpe's CAPM formulation was founded on the mean-variance paradigm and an understanding of equilibrium: on balance, all rational, risk-averse, profit-seeking investors would hold the market portfolio in combination with the riskless asset. It follows that the market portfolio is the value-weighted portfolio of the investment universe, satisfying the average risk-aversion of all investors. Consequently, the CAPM asserts that any asset is expected, under equilibrium conditions, to earn the investor a profit in excess of the risk-free rate that is linearly related to its market beta (refer Equation 2.5).

**Equation 2.5: Capital Asset Pricing Model (CAPM)**

$$E(R_i) - r_f = \beta_i E[R_M - r_f]$$

where  $E(R_i)$  is the expected return of an asset,  $i$ ;

$E[R_M - r_f]$  is the expected market risk premium

$r_f$  is the risk-free rate; and

$\beta_i$  is the market beta of asset,  $i$ , and describes the sensitivity of the asset to market returns.

The CAPM formulation of risk and return provided investors with an understanding of risk: systematic, market-related risk and diversifiable, residual risk. But the MPT framework of

which it is part is ill suited to competing professional fund managers, looking to distinguish themselves by taking opportunities of dis-equilibrium within their investment universe. In the introduction to their book on the subject of active management, (Grinold and Kahn, 2000) describe active management as the poor relation of modern portfolio theory: active management uses the equilibrium and optimisation concepts of Markowitz and Sharpe but pursues the absence of equilibrium. Active managers actively seek non-equilibrium returns and must manage the uncertainty of their forecasts accordingly. In fact, active return forecasts are commonly referred to as “alphas” because they attempt to forecast returns that are residual to those anticipated by the CAPM.

In their replacement of the riskless asset with the benchmark portfolio, Goetzmann *et al.* (1996) maintained the mean-variance concept of uncertainty and trade-offs and, as such, the active management problem is defined as the optimisation of active return subject to the minimisation of active risk for a particular level of active risk aversion. The objective function for this optimisation is represented by Equation 2.6.

#### **Equation 2.6: Active management optimisation problem**

$$\text{Maximise } \mathbf{w}_a' \boldsymbol{\alpha} - \frac{1}{2} \lambda \mathbf{w}_a' \boldsymbol{\Sigma} \mathbf{w}_a$$

where  $\boldsymbol{\alpha}$  is an  $n \times 1$  vector of forecast active returns for each of  $n$  assets and  $\lambda$  is the active risk aversion parameter ( $\lambda > 0$ ). In practice, the forecasts ( $\boldsymbol{\alpha}$ ) reflect the present cross-sectional view of investment opportunities for a particular investment horizon. These views and the risk model ( $\boldsymbol{\Sigma}$ ) on which the optimisation is based will change with time and new information. Every change in the alpha vector and the estimates of the covariance matrix necessitates a change in the portfolio's weights and portfolios are, where practical and cost effective, rebalanced and adjusted to reflect these changes.

As with mean-variance optimisation, in the absence of portfolio constraints, the weights that solve this objective function in Equation 2.6 have unique relative values that are scaled by the risk-aversion parameter. The risk-reward characteristics of each portfolio for each active risk can be plotted as an efficient frontier in an active risk and active return space, representing various active-risk tolerances from zero (the passive benchmark portfolio) to greater active risk tolerances.

In practice, the role of the risk-aversion parameter is replaced by a target active risk,  $\sigma_A$ . Thus the optimisation finds the highest expected active return,  $(\mathbf{w}'\boldsymbol{\alpha})$  with the required amount of active risk (i.e.  $\mathbf{w}'\boldsymbol{\Omega}\mathbf{w} = \sigma_A$ ). For any given active risk target, there is a well-known, unique solution (refer Equation 2.7) to this problem:

**Equation 2.7: Unconstrained optimal active weights for a given target active risk**

$$\mathbf{w}_a^{unconstrained} = \sigma_A \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{\alpha}}{\sqrt{\boldsymbol{\alpha}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\alpha}}}$$

where  $\sigma_A$  is the target active risk of the portfolio i.e. the particular level of active risk for which the optimisation is solved and  $\mathbf{w}_a^{unconstrained}$  is a vector describing the active weights of the optimal unconstrained portfolio.

Equation 2.7 shows that the size and sign of the active weight of any particular stock is directly related to the size and sign of the forecasted active return when the portfolio is unconstrained. The target active risk magnifies the extent of this active weight while the risk in the denominator has the opposite effect. Thus a positive excess-of-benchmark return expectation (relative to the other forecasts) for a particular asset would lead a rational, unconstrained and optimal fund manager to a positive active position in that same



asset. The greater the target active risk (i.e. the lower the risk-aversion) of the investor, the greater the active positions in their portfolio will be. Conversely, the extent of this active position is reduced by the uncertainty of the asset's prospects.

### **2.1.1 The Fundamental Law of Active Management**

In practice, the optimisation of active return forecasts relative to active risk resulted in more diverse portfolios than the traditional mean-variance optimisation but was subject to some of the same criticisms: the portfolio weights that solve the optimisation are very sensitive to forecast errors (Jobson and Korkie, 1981; Jorion, 1992; Broadie, 1993; Michaud, 1989; Best and Grauer, 1991). The resulting portfolios tend to be “error maximisers” (Michaud, 1989) rather than return maximisers.

Grinold (1994) responded to the criticisms of portfolio optimisation procedures as “alpha eaters”: that good excess return forecasts are distorted by the standard portfolio optimisation procedures and that the resulting portfolios consequently generate less of the excess-of-benchmark (“alpha”) which they ought to. In developing the “Fundamental Law of Active Management”, Grinold (1994) showed that, if forecasts are treated as a product of residual volatility (which was assumed to be independent across stocks) times skill (as measured by the information coefficient (IC)) times a standardised score (refer Equation 2.8), the resulting optimised portfolios will not exhibit the same bias toward low residual stocks. In this alpha-generation formulation, Grinold (1994) offered practical instruction on how to treat a stock tip, a buy/sell list or a series of multiple forecasts as a score, translating these scores into an alpha (forecast) that “won’t get eaten” by an active management optimisation.

**Equation 2.8: Alpha generation (Grinold, 1994)**

$$\alpha_i = IC \cdot \sigma_{e,i} \cdot S_i$$

where

IC is the Information Coefficient: the forecasting skill of the manager or the expected correlation between the manager's forecast active return ( $\alpha_i$ ) and their subsequent realised active return;

$\sigma_{e,i}$  is the residual risk of asset,  $i$  and

$S_i$  is the standard normal score for each asset which represents the relative view on the cross-section of assets.

Grinold (1994) demonstrates that, when alphas are derived in this way (Equation 2.8), the unconstrained optimal portfolio that is constructed using these alphas will have an expected return equal to the manager's forecasting skill (IC) times the breadth with which that skill is applied multiplied by the active risk of the portfolio (refer Equation 2.9). The fundamental law thus states that the greater the manager's forecasting skill, the more broadly that skill is applied (by taking multiple active positions) and the more active risk the manager takes, the greater the expected return of the optimal unconstrained portfolio.

**Equation 2.9: Fundamental Law of Active Management (Grinold, 1994)**

$$E(R_a) = \sigma_a \cdot IC \cdot \sqrt{N}$$

where

$E(R_a)$  is the expected active return of the portfolio;

$\sigma_a$  is the active risk of the portfolio; and

$N$  is “breadth”: the number of independent positions (alphas) in the portfolio.

### 2.1.2 The transfer coefficient

Grinold (1994) failed to address the alpha “eating” effect of constraints on portfolios (and optimisers) and omitted a consideration of off-diagonal elements of the covariance matrix: Grinold (1994) worked on the assumption that the residual (not market-related) returns were uncorrelated among assets. Clarke, De Silva and Thorley (2002), Clarke, De Silva and Thorley (2005), and Clarke, De Silva and Thorley (2006) extended the work of Grinold (1994), deriving similar relationships as in Grinold (1994) but allowing for interaction terms among residual risks.

Clarke *et al.* (2002) introduced the concept of a transfer coefficient. The authors illustrate the loss of excess risk-adjusted performance that can result from portfolio constraints, particularly the long-only constraint preventing short sales. The authors use the transfer coefficient (TC) to quantify the extent of this loss. The transfer coefficient is defined as the cross-sectional correlation of the risk-adjusted forecasts across assets and the risk-adjusted active portfolio weights in the same assets. A transfer coefficient of one implies that there is no “friction” between the manager’s forecast returns and the construction of the investment portfolio. A transfer coefficient less than one implies a loss of information between the manager’s forecast returns and the construction of the investment portfolio based on these forecasts.

**Equation 2.10: Definition of Transfer Coefficient (TC)**

$$TC = \text{Correlation}\left(w_a \sigma, \frac{\alpha}{\sigma}\right)$$

where  $w_a \sigma_e$  is a vector of risk adjusted active weights ( $w_{a,i} \sigma_i$ ) and  $\sigma_i$  is the risk for each asset.

In this way, the transfer coefficient essentially measures the manager's ability to invest in a way that is consistent with their current, relative views on the assets in their investment universe. The unconstrained optimal portfolio described by Equation 2.7 would have a transfer coefficient of one by definition. Any inconsistency in implementation, including the compliance to mandated limits on investment weights, will reduce the transfer coefficient below one.

To accommodate the constrained portfolio manager and incorporating the full covariance matrix, Clarke *et al.* (2006) developed the "Generalised Fundamental Law of Active Management". As in Equation 2.9, this generalised law (Equation 2.10) describes the relationship between the expected return of the portfolio and the manager's skill and active risk budget. But in the generalised form, this relationship includes the detrimental effect of constraints that impose sub-optimal portfolio construction by multiplying by the transfer coefficient.

**Equation 2.11: The Generalised Fundamental Law of Active Management**

$$E(R_a) = \sigma_a \cdot TC \cdot \sqrt{\alpha^T \Sigma^{-1} \alpha}$$

Equation 2.11 is written in full covariance form but represents the same relationship as Equation 2.9. The only differences between the generalised form of the fundamental law (Equation 2.11) and the original fundamental law (Equation 2.9) is the incorporation of interaction terms in the covariance matrix and a TC which is not necessarily one. In fact, Equation 2.9 can be derived by substituting a TC of one and the alpha generating formula of Equation 2.8 into Equation 2.11.

Of course, the definition of the transfer coefficient in Equation 2.10 was redefined by Clarke *et al.* (2006) to incorporate the full covariance matrix (refer Equation 2.12). Once again, substituting the alpha generating formula in Equation 2.8 into Equation 2.12 would result in the same definition of TC as described in Equation 2.10, demonstrating the consistency of this full covariance definition with the former definition.

#### **Equation 2.12: TC under full covariance assumptions<sup>6</sup>**

$$TC = \frac{\alpha' w_a}{\sqrt{\alpha' \Sigma^{-1} \alpha} \sqrt{w_a' \Sigma w_a}}$$

Calculating the transfer coefficient can assist the portfolio manager to assess the efficiency of their implementation – how consistent are the active weights in the portfolio with the alpha forecasts and risk matrix? More importantly in the context of this thesis, the transfer coefficient enables an assessment of the inefficiencies introduced by the mandated

---

<sup>6</sup> When active weights are zero i.e. the portfolio is the same as the benchmark, this ratio is not defined. However, clearly, when no active risk is taken the TC is meaningless.

restrictions of the portfolio. It is this latter application of transfer coefficient that can assist fund sponsors and mandate authors in their understanding of the impact of constraints on the portfolio's performance.

## **2.2 THE LONG-ONLY CONSTRAINT**

These authors of Clarke *et al.* (2002) pioneered the use of short extension products that allow for modest short positions and have been shown to improve the transfer of manager skills to the fund while maintaining a net long investment. Several empirical studies, such as Clarke, De Silva, and Sapra (2004) and Martielli (2005), have demonstrated the impact of investment constraints on the performance of the portfolio, using the transfer coefficient to quantify the extent of the value lost in implementation. These empirical studies show that, of all the typical mandated fund constraints, the prohibition on short positions in a portfolio accounts for the greatest loss of value between forecasts and portfolio implementation.

Short extension products have proved to be one solution to this problem in international markets. The typical, fully invested, equity portfolio in a pension fund's overall investment portfolio is constrained to be "long-only" (i.e. hold no short positions in any assets) and fully invested (i.e. 100% of the fund is invested without gearing). Short extension products offer this segment of the market, hardest hit by the inefficiency of size and investment constraints, an opportunity to be 100% invested with only a partial relaxation of the short constraint. For example, a so-called 130/30 portfolio allows for a maximum of R30 of every R100 invested to be held in short positions and R130 to be invested long.

This so-called “short extension” can be used to increase the risk and gearing of a previously long-only fund. However, in keeping with the original intention of this investment product and in the context of this thesis, short extension has as its purpose to increase the transfer of forecast information from the fund manager to the fund for the same level of active risk as its long-only counterpart.

Analytic Investors<sup>7</sup> (chaired by Roger Clarke) are considered to be the pioneers in short extension products and have been managing short extension products on the S&P 500 and Russell 1000 since 2002. Many others have followed suit, resulting in an increasing demand for these partially short products amongst international pension investors.

The empirical studies of Clarke *et al.* (2004) and Martielli (2005) amongst others, have shown the improvement to the transfer coefficient offered by these short extension products, confirming the relationship between ultimate performance and both the accuracy of the fund manager’s forecasts and their ability to appropriately transfer these forecasted investment views to the portfolio. These studies also show the extent to which high market concentration in large capitalisation stocks and increased target active risk exacerbate the detrimental effect of the long-only constraint on the transfer coefficient and ultimately the performance of an otherwise well-managed fund.

In a small and highly concentrated equity market such as South Africa where roughly half of our equity index is comprised of only five securities, the effect of constraints on investment weights (such as the long only constraint) can be even more detrimental than

---

<sup>7</sup> <http://www.aninvestor.com/index.asp>

has been shown in international literature, which is typically based on indices of 500 or more securities. Short extension products have only recently (2009) been available to South African pension funds and as yet no published track record exists for funds invested in this way. In considering these funds as a viable alternative to long-only investment, South African investors and pension fund sponsors have been challenged to reconsider the restrictions they typically mandate on securities in their investment universe. Chapter 3 addresses some of these concerns.

Clarke, De Silva, Sapra and Thorley (2008) concerned itself with the quantitative boundaries and guidelines required by mandate authors to accommodate short extension funds but the analysis in this article applies to all investment weight restrictions and, as such, provides useful models for mandate authors of actively managed equity funds in general. Clarke *et al.* (2008) provides a generalised mathematical model of the distribution of optimal unconstrained investment weights in every asset across various forecasts. This derivation clarifies the relationship between the extent of the optimally required short positions in a portfolio and a) the number of assets in the benchmark, b) the concentration of the benchmark, c) the overall risk of each asset, d) the overall level of correlation among assets and e) the target active risk.

The premise of Clarke *et al.* (2008) is that investment views at any point in time on any particular stock can be described as a random normal variable. Consequently, the optimal unconstrained investment weights are also random normal with an expected value that is a function of the benchmark, the fund's risk budget and market risk conditions. The derivations of the authors in this section have been extended to the South African Equity environment in Chapter 6.



## 2.3 CONCENTRATION

Strongin, Petsch and Sharenow (2000), introduced the detrimental effect of using concentrated market indices as benchmarks. Curious about the failure of large-cap fund managers to deliver superior performance, the authors argued that the problem was not one that could be solved by correcting for a “size” factor or by improved forecasting or stock selection. Instead, they argue, the undiversified indices which these fund managers use as benchmarks are to blame. Strongin *et al.* (2000) demonstrates that concentrated market capitalisation-weighted indices are poorly designed for active management because they present a passive investment with a high degree of stock-specific risk. “The concentration of stock-specific risk in the large-capitalization indexes is so large that the indexes are taking more stock-specific risk than the portfolio manager. As a result, the portfolio manager’s performance relative to the benchmark is driven by the index rather than by the skill of the portfolio manager.”

Strongin *et al.* (2000) derive a measure of concentration that is essentially the inverse of the Herfindahl–Hirschman Index (HHI). The “effective number of stocks” (refer Equation 2.13) in an index is the number of stocks which, equally weighted, would provide the same level of diversification as the index.

### Equation 2.13: Effective number of stocks in an index/benchmark

$$\tilde{N} = \frac{1}{\sum_i w_{b,i}^2}$$

where  $\tilde{N}$  is the effective number of stocks in an index.

The authors show that, if the weight of a stock in an index is greater than  $\frac{2}{N-1}$ , it creates more stock-specific risk than it diversifies away.

Following on Strongin *et al.* (2000), Kruger and Van Rensburg (2008) explored the risk implications of the concentration inherent in the established FTSE/JSE equity indices in 2002: the ALSI, Shareholder Weighted Indices<sup>8</sup> (SWIX), Capped Indices (CAPI) and various combinations of the Resources (RESI) and Financial and Industrial (FINDI) indices. The authors provide a good review of the advantages and disadvantages of the use of these indices as investment benchmarks for professional fund managers. In particular, they compare their levels of concentration and find the ALSI to be the most concentrated. Capped indices and indices that provide lower exposure to resource stocks in general, provide an obvious remedy to the concentration of South Africa's equity market in two big resource stocks. The FTSE/JSE has since launched an equally weighted index (ETOP40) on the largest stocks, which will further address the concentration issue.

To some extent, these solutions are artificial. Specially designed benchmarks provide a yardstick against which to assess active performance against the same benchmark. In this way fund sponsors can carve out focussed management styles and less concentrated mandates. Capping or reducing the weight in large stocks in the fund's benchmark enables a less concentrated carve-out of the investment mandate. But Sharpe (1991) points out that the average actively managed fund (i.e. capitalisation weighted) cannot,

---

<sup>8</sup> The shareholder-weighted index series attempts to represent only the domestic investors in each index to which this weighting is applied. The JSE provides a shareholder weighted figure from the authorised Central Securities Depository (CSD) for the electronic settlement of all financial instruments in South Africa.

before costs, deliver performance different to the average passively managed rand, by virtue of basic arithmetic. Consequently, when considering the average invested rand, which represents the wholesale equity landscape, only a capitalisation-weighted index will do and, as such, South Africa's market remains concentrated despite these bespoke benchmarks.

The creation of the SWIX specifically had the domestic pension fund investor in mind. This index excludes foreign holdings in dual listed stocks and, because of the dominance of the dual listed resource stocks, thereby provides a less concentrated representation of the opportunity set of equity investment to domestic pension funds compared to the ALSI.

For the most part, Kruger and van Rensburg (2008) and Strongin *et al.* (2000) are concerned with the lack of diversification in the various benchmarks and the contribution of that lack of diversification to the risk of funds that use them as benchmarks. While issues of diversification are certainly important, particularly in a small, concentrated investment universe such as South Africa's, this thesis is concerned with the application of portfolio construction within this concentrated environment. Chapter 3 explores the concentration of the South African equity market and its implications for active fund management in greater detail.

## **2.4 CROSS-SECTIONAL VARIANCE OF RETURNS**

Sapra (2008), Ankrum and Ding (2002) and De Silva *et al.* (2001), amongst others, point out that modern portfolio theory is founded on an understanding and an emphasis on time series or longitudinal volatility. Both Markowitz's ground-breaking 1952 introduction of a

mean-variance paradigm and the Fundamental Law of Active Management, “Alpha is IC times volatility times score” require a time-series estimation of risk. This estimation usually involves successive periods of realised or forecast performance.

But when active fund managers decide on how to allocate the finite pool of assets under their management among various investments, it is the cross-sectional dispersion of expected returns that is required in order to provide them with a reasonable opportunity for expressing relative preferences. As Sapra (2008) points out, if all securities had perfect correlations with each other, there would be no cross-sectional dispersion in their returns and therefore no way for an active manager to achieve excess-of-benchmark performance or incur any active risk. Without cross-sectional volatility, the active fund manager would be unable to deliver performance which was in any way distinct from their benchmark or any of their competitors.

Solnik and Roulet (2000) explored the nature of cross-sectional correlation in the context of discerning the relationships between global markets as part of the global allocation decision. Traditionally, longitudinal or time-series data would be used to estimate correlation between the various global markets’ performances. Typically, this would be done using a rolling 60-month window of simultaneous returns across world markets.

Solnik and Roulet (2000) point out that there are several problems with this type of measurement of global correlations. Firstly, the measurement of longitudinal relationships is unconditional i.e. it assumes that the relationships between global markets do not change over time and neither do the distributions of the returns of these markets. Furthermore, each subsequent estimate is highly dependent on the previous estimate and

is therefore also resistant to changes in the global market relationships. Although various weighted measures and autoregressive solutions exist for this problem, they either require a large number of estimations, a diagonal matrix or an ad hoc system of time-series weights that do not necessarily solve the dependence problem.

Solnik and Roulet (2000) propose that a cross-sectional, “instantaneous” measure of correlation is more appropriate and more capable of detecting changing trends in market relationships. These authors find that the cross-sectional method they proposed is more dynamic and that changes in this measure helped predict changes in the longitudinal estimation better than a time series approach would.

#### **2.4.1 Importance to ex ante active management**

Changes in the volatility, either longitudinal or cross-sectional, are of concern to investors and asset management selectors as they signal changes in the professional money management environment. Bernstein (1998) observed an apparent decreasing dispersion over time (from 1969 to 1997) of fund returns in the US as well as the decreasing spread in the top performing portfolios relative to the benchmark. In this article, Bernstein puts forward two hypotheses: a) markets have become increasingly competitive and therefore it is increasingly difficult to significantly outperform the competition or b) risk management and the fear of being “wrong and alone”<sup>9</sup> has created convergent investments across portfolios without the concentration of risk required to produce star performance. Bernstein provides some tantalising but exploratory empirics into the latter and this

---

<sup>9</sup> A phrase coined by Mark Kritzman in his 1998 article, "Wrong and Alone", Economics and Portfolio Strategy.

hypothesis would provide a fascinating future research project into the behaviour of professional money managers. However, in this dissertation it is the former hypothesis that is of interest.

Bernstein (1998) uses baseball as an analogy of what happens to star performers when the variation in the performance of all athletes wanes. In 130 years of baseball data, the batting averages have hardly varied. However, the standard deviation around that long-term mean has diminished suggesting the convergence of batting scores. The consequence of this diminishing variation has been the disappearance of the “.400 star hitter”. While new records continue to be set in individual sports, stars within team sports can disappear because of improved defence and competitive strategies.

Bernstein (1998) likened this to the field of professional investment management where he mused that the US stock market may have become increasingly efficient. Bernstein went so far as to show the waning of the spread in the performance of Warren Buffet's portfolio (the investment management equivalent of a star athlete!) relative to the S&P 500 up to this 1998 article. Bernstein argues that even a skilled, star investor such as Buffet, with no regard for benchmarks, has found his portfolio's performance spread relative to the S&P 500 decreasing over time.

However, subsequent to 1997, Ankrum and Ding (2002) observed an increase in market volatility and noticed that the cross-sectional volatility of the market was also increasing. Ankrum and Ding (2002) makes the connection between longitudinal or time-series volatility and cross-sectional volatility, deriving a mathematical link between overall market volatility,

sector volatility and intra-sector, cross-sectional volatility. (Their results hold whether sectors are considered or not.)

The authors, mindful of the thinking of the time (2001) that equity market volatility was on the increase, questioned the fact that active portfolio returns were increasing in their dispersion in the same way and suggest that both are a function of increasing cross-sectional volatility, which is, after all, the environment in which an active manager is active.

Ankrum and Ding (2002) postulates a number of possible reasons for the observed increase in cross-sectional volatility at the time: the trend towards smaller, more focussed companies; companies listing their equity earlier in the company's life cycle than has been the case historically; the influence of day-traders; less expensive trading and greater financial innovation; increases in financial leverage; and shocks to the discount rates applied to company valuations. Certainly, the South African and other emerging/developing markets are subject to similar trends and practices and the effects may well be seen in our changing equity volatility.

De Silva *et al.* (2001) observed the highest recorded cross-sectional volatility in the US in 1999 (increasing from the last quarter of 1998 to 2000). In this article, the authors speculate as to the reason for this unprecedented dispersion, which they say economic historians would suggest is attributable to fundamental changes in the competitive advantages among companies. The authors suggest a new factor in security returns such as the impact of technology or alternatively a purely idiosyncratic event as possible culprits. The focus of De Silva *et al.* (2001) is to remedy the view of ex post performance in the light of varying cross-sectional dispersion.

Ten years before the De Silva *et al.* (2001) article, Sharpe (1991) addressed the basic arithmetic of the ex post performance measurement of portfolios without any market efficiency assumptions or statistical simplifications required. Sharpe (1991) points out that when we consider the performance of various managers at any point in time in an equally weighted manner in order to assess their relative success, we incur various biases and mathematical violations.

Firstly, Sharpe (1991) makes the point that the average actively managed dollar (i.e. capitalisation weighted) cannot, before costs, deliver performance different to the average passively managed dollar. It follows that, after costs, capitalisation weighted investment dollars must underperform passive dollars when actively managed. Although mathematically obvious, Sharpe points out several performance monitoring practices that violate these basic tenets.

Firstly, when active portfolios that are compelled to hold cash are compared to equity-only benchmarks, the average dollar so invested will underperform the benchmark in good times and over perform in bad times. Another classic case of inappropriate averaging is the use of only the surviving portfolios as a performance comparison. Survivorship bias omits the dollars that were transferred to another investment portfolio during the performance period in question.

Sharpe also points out the potential for a small-cap bias when equally weighted manager averages are compared to the market as a whole. If portfolios of smaller fund sizes show a preference to smaller capitalisation stocks as is typical, the “average” portfolio will



overweight smaller portfolios, thereby overweighting small caps in the “average” performance.

Small-cap bias is one of the reasons that peer-group comparisons are ill advised. The most compelling argument against peer benchmarks is that since capitalisation-weighted active portfolios will, by virtue of pure mathematics, underperform passive portfolios after costs, a peer group benchmark is necessarily an underperforming benchmark relative to a passive capitalisation-weighted index.

De Silva *et al.* (2001) extends the Sharpe (1991) argument regarding fund returns and their relationship to stock returns by showing that the value-weighted cross sectional dispersion of security returns must necessarily be related to the value-weighted cross section dispersion of active portfolio returns. Although diversification within portfolios would necessarily mean that portfolio returns are less dispersed than security returns, the relationship between the cross-sectional variations in each would hold.

De Silva *et al.* (2001) derives a mathematical relationship between security return dispersion, market return and idiosyncratic risk (refer Equation 2.14) to inform the expectation of dispersion among securities. Relying on some simplifications, specifically an equally-weighted market portfolio and a common distribution of idiosyncratic risk across stocks at any given time<sup>10</sup>, this derivation shows that expected cross sectional dispersion

---

<sup>10</sup> The derivation has the CAPM segmentation of return as its starting point. Without a common distribution of idiosyncratic return from which to draw each stock's idiosyncratic return, the cross-sectional distribution of stock returns would be complex and non-normal.

increases with a) increasing dispersion among stock betas, increasing idiosyncratic risk across stocks and market performance that differs increasingly from the risk free rate (either positively or negatively).

**Equation 2.14: Relationship between market dispersion and security dispersion**

$$E(D_t^2) = \sigma_{\beta,t}^2 (R_{M,t} - r_{f,t})^2 + \sigma_{e,t}^2$$

where

$E(D_t^2)$  is the expected (equal-weighted) cross-sectional dispersion among securities over an investment period,  $t$ ;

$\sigma_{\beta,t}^2$  is the cross-sectional variance of stock betas around one over this same period;

$R_{M,t} - r_{f,t}$  is the excess-of-risk-free rate return of the (equal-weighted) market over the investment period,  $t$ ; and

$\sigma_{e,t}^2$  is the idiosyncratic risk, which is assumed to be the same for all stocks over this period.

De Silva *et al.* (2001) extends this derivation to illustrate the relationship between portfolio performance dispersion and overall market performance (refer Equation 2.15). It follows that, since both portfolio and security dispersion are related to idiosyncratic risk, portfolio and security dispersion must also be related to each other.

**Equation 2.15: Relationship between portfolio dispersion and security dispersion**

$$E(D_p^2) = \sigma_{\beta_p}^2 (R_M - r_f)^2 + \sigma_{\varepsilon_p}^2$$

where

$E(D_p^2)$  is the expected (equal-weighted) cross-sectional dispersion among portfolios of securities;

$\sigma_{\beta_p}^2$  is the cross-sectional variance of portfolio betas (which, if stocks are selected randomly will be  $\frac{\sigma_{\beta,t}^2}{n}$ ); and

$\sigma_{\varepsilon_p}^2$  is the idiosyncratic risk of portfolios of securities (which, under the assumption of zero correlation among idiosyncratic security returns is equal to  $\frac{\sigma_{\varepsilon}^2}{n}$  where  $n$  is the number of stocks randomly selected in the portfolio).

Yu and Sharaiha (2007) develops the findings of De Silva *et al.* (2001) by looking at cross-sectional dispersion as it informs the ex ante risk budgeting decision. The authors argue that, in an active management context, active positions must be taken in a portfolio in such a way that the extent of the position is justified by the size and reliability of the forecasts/alphas. As such, Yu and Sharaiha (2007) presents cross-sectional volatility as a method for measuring active management opportunities or, what the authors term, the “alpha-granularity” of markets at any point in time.

In an attempt to address the risk-budgeting decision with particular reference to top-down and bottom-up allocation, Yu and Sharaiha (2007) introduces the orthogonal relationship between asset allocation dispersion and stock-selection dispersion, collectively constituting the total return dispersion at any given time. This method enables a

comparison between these two components of the risk budgeting decision within the common basis of the alpha-opportunities available at the time.

The authors go on to recommend that such an analysis be compared to the existing dispersion in the active weights of a portfolio to assess whether the alpha-risk budget is being invested with similar ratios. A ranked correlation of the dispersion within sectors for a portfolio against the dispersion within sectors for a benchmark gives an indication of the closeness of the risk budgeting decision with the alpha opportunities in the market, although portfolios should be constructed in anticipation of future alpha opportunities.

In Yu and Sharaiha (2007), the authors show by derivation that the un-weighted cross-sectional dispersion is directly proportional to the returns on a dollar-neutral<sup>11</sup> investment. In this way, the authors propose the use of cross-sectional volatility in a hedge fund context as a perfect hindsight performance benchmark. They propose that, when assessing the skill of the portfolio manager to allocate their risk budget, the dispersion of the funds' returns be compared to the realised dispersion of the appropriate investment universe: the higher the correlation between these two, the more appropriate the risk budgeting has been.

Finally, Yu and Sharaiha (2007) derive the mathematical relationship showing that the expected (weighted) cross-sectional volatility is equal to a) the differential between the weighted average stock volatility and market volatility and b) the dispersion among expected stock returns. The authors continue, using a multivariate diffusion approach

---

<sup>11</sup> Equal quantities of long and short investments with a net value of zero.

as return generating processes, and derive a similar relationship to that of Sapra (2008) (refer Equation 2.16): cross-sectional volatility (unweighted) is proportional to the average stock return volatility times 1 minus the average correlation among stocks.

**Equation 2.16: The relationship between cross-sectional volatility, average volatility and average correlation**

$$\sigma_{cs}^2 \propto \overline{\sigma_t^2} (1 - \bar{\rho})$$

where

$\sigma_{cs}^2$  is the expected weighted cross-sectional volatility;

$\overline{\sigma_t^2}$  is the average security volatility (longitudinal); and

$\bar{\rho}$  is the average correlation between pairs of securities.

Yu and Sharaiha (2007) goes so far as to argue that falling dispersion (decreasing  $\sigma_{cs}^2$ ) is a business risk to active managers, particularly managers of long/short portfolios. As such, one of the article's suggested remedies for the active portfolio management business is to hedge their revenue against falling cross-sectional volatility by employing the use of variance swaps and/or dispersion trades.

Extending the work of De Silva *et al.* (2001), Sapra (2008) looks at the loss of efficiency caused by unexpected (i.e. incorrectly forecast) changes in cross-sectional dispersion. In this article, an expression relating the estimation error in cross-sectional volatility to the realised active risk is derived as follows:

**Equation 2.17: The relationship between unexpected change in cross-sectional volatility and realised active risk**

$$\frac{\sigma_A}{\widehat{\sigma}_A} = \frac{\sigma_{cs}}{\widehat{\sigma}_{cs}}$$

where

$\sigma_{cs}$  and  $\widehat{\sigma}_{cs}$  are the realised and forecast cross-sectional standard deviation of the investment universe respectively; and

$\sigma_A$  and  $\widehat{\sigma}_A$  are the realised and target active risk of a portfolio.

Sapra (2008) shows that this “shock” to cross sectional risk implies a decrease in the information ratio of a manager’s realised performance – a ratio which is used as indicator of manager skill to potential investors and existing clients.

In Gorman, Sapra and Weigand (2010), the findings of Sapra (2008) are extended with an investigation into the predictability of cross-sectional volatility using, in particular, the VIX<sup>12</sup> (implied volatility index). Other authors such as Ratner, Meric and Meric (2006), Clarke, De Silva and Thorley (2010) and Ang, Hodrick, Xing and Zhang (2006) have attempted to use cross-sectional dispersion as a predictor of stock level performance and volatility. In Chapter 4, the focus is on the implications of varying cross-sectional dispersion for the mandate setting and monitoring process rather than forecasting volatility or returns.

---

<sup>12</sup> The South African equivalent is the SAVI.

### **2.4.2 Importance to ex post performance assessment**

An important component of effective professional management, in any arena, requires regular performance monitoring of those who are being managed. Good and reasonable performance monitoring, one could argue, requires the measurement of success in a way that is as objective and as fair as possible given the particular management environment. This measurement ought to be mindful of the instructions, constraints and mandates given to those whose performance is being assessed as well as the conditions under which the success (or lack thereof) was achieved.

Professional fund management is no different. The decision makers of a pension fund, for example, may instruct several fund managers with different investment mandates. The fund managers, in turn, manage a finite pool of assets by investing, avoiding or disinvesting in a variety of value-changing assets in the hope of adding value to their portion of this pension fund within the required mandates and constraints. The performance of these investment portfolios (which is essentially also the performance of their managers) are very closely monitored by the pension fund advisors and the awarding of large and profitable investment contracts relies on the satisfactory and competitive performance of these portfolios.

Chapter 5 aims to contribute to the fairness and appropriateness with which the performances of these portfolios are monitored, particularly over multiple periods, mindful of the market environment over which the performance is sampled.

### 2.4.2.1 Risk-adjusted performance measurement

With increasing transparency and measurement standards (such as GIPS®) the calculation and fair comparison of changes in fund values over any period (i.e. the “return”) is achievable. The Modern Portfolio Theory (MPT) framework first developed by Markowitz, Sharpe, Tobin and others introduced the importance of considering risk as well as return when assessing performance. In particular, MPT introduced the longitudinal risk of individual assets and the efficiency available through understanding the risk relationships between assets. Measures such as the Sharpe ratio (refer Equation 2.18) have influenced the objectives of the modern professional fund manager who is now required to provide not just return but risk-adjusted return to the fund. Citing MPT investors would argue that profit delivered at a lower risk is more impressive and more efficient than the same profit delivered with greater volatility.

#### Equation 2.18: Sharpe ratio (Sharpe 1966)

$$\frac{E(R_p - r_f)}{\sqrt{\text{Var}(R_p - r_f)}}$$

where

$E(R_p - r_f)$  is the expected return of the portfolio in excess of the risk free rate; and

$\text{Var}(R_p - r_f)$  is the variance of these excess-of-risk free rate returns and is the same as  $\text{Var}(R_p)$ .

The now commonly used ratio introduced in Sharpe (1966) was expressed in ex ante terms however it is commonly applied retrospectively to fund performance and, as such, is used in an attempt to deduce the existence of superior “skill” on the part of the fund



manager using historical evidence. Applied ex post, this statistic attempts to discern whether the mean of the distribution of the fund's performance in excess of the risk free rate significantly differs from zero, using the sample mean and standard deviation as an estimate. In this way, the Sharpe ratio is similar to a t-statistic for the hypothesis that a fund is delivering significantly more profit than its riskless alternative (refer Equation 2.19).

**Equation 2.19: t-statistic testing the hypothesis of superior risky performance**

$$t = \frac{\overline{(R_p - r_f)} - 0}{\frac{s_p}{\sqrt{T}}}$$

where

$\overline{(R_p - r_f)}$  is the realised time-series average return of the portfolio in excess of the risk-free asset;

$s_p$  is the estimated time series standard deviation of this excess performance; and

$T$  is the number of independent time periods over which the performance is measured (akin to sample size).

The Sharpe Ratio is one of many different ways to measure ex post risk-adjusted performance and the methodology would typically be agreed upon by both parties in advance. Suffice it to say that modern fund managers are required to manage both the hope of an increase in fund value as well as its uncertainty and they expect to have their services assessed accordingly after the fact.

#### **2.4.2.2. Benchmark-adjusted performance measurement**

More recently, professional fund management necessitated another change in performance measurement. While investors remain sensitive to the profit uncertainty over time of their investments, the professional fund manager's role is arguably more about their ability to glean profit from their investment universe relative to a pre-determined, "passively" managed, investable benchmark. Hence the term "active" management and the revised framework for investment decision-making as developed by Grinold and Kahn (1994) and others. In this professional management context, fund managers are neither rewarded nor punished for the profits generated from a passive<sup>13</sup> investment alternative to their own mandate since this passive return can be earned without the services of a professional fund manager. For example, a fund manager tasked with investing in South African large capitalisation equity will likely have their fund's performance benchmarked against the FTSE/JSE Top 40 Index (ALSI 40). Consequently, the change in their fund's value in every period will be corrected for the performance of this index in terms of both profit and risk.

Grinold and Kahn (1994) present an alternative ratio more suited to the active management framework: the Information Ratio (refer Equation 2.20). This ratio is also

---

<sup>13</sup> This argument assumes the possibility of a costless indexed investment such as an exchange traded fund (ETF) that can be bought and held without any transacting required. In reality, passive funds require substantial management when there are distributions, corporate actions, cash flows and changes to the membership of the index. The more tailor-made, non cap-weighted benchmarks require even more transacting to keep the passive fund's composition in line with the composition of the index. However, the systematic, rules-based nature of these funds makes them sufficiently passively managed to provide a comparative performance with a fund in which the manager intervenes at any time to implement their investment view.

applied ex post to realised fund performance in order to assess whether the fund manager has significantly contributed to the fund's performance beyond that of a passive fund.

### Equation 2.20: Information Ratio<sup>14</sup>

$$\frac{\overline{(R_p - R_b)}}{s_a}$$

where

$\overline{(R_p - R_b)}$  is the time series average of realised excess-of-benchmark or active returns, and

$s_a$  is the longitudinal standard deviation of the same realised excess-of-benchmark returns  $\overline{(R_p - R_b)}$ .

Monitoring the fund manager's performance in this way is considered fair and objective because the measurement takes into account both the tools at the fund manager's disposal as well as the conditions prevailing at the time of measurement. For example, a large cap manager cannot be expected to benefit from a small cap boom during their investment horizon and so the comparison of the large cap manager's return to an appropriate large cap benchmark fund is more appropriate than measuring the fund's return in isolation. Statistically speaking, subtracting the benchmark return from a series of fund returns is an attempt at correcting the time series of fund returns (the only evidence of fund manager skill) for the first moment, the mean. In fact, the Information Ratio in Equation 2.20 is very much like a t-statistic for a paired t-test, testing the hypothesis that

---

<sup>14</sup> As with the Sharpe Ratio, the Information Ratio has an ex ante definition as well: the ratio of expected active return divided by the ex ante estimate of active risk.

the mean of the fund's returns differs significantly from the mean of its benchmark's returns (refer Equation 2.21).

**Equation 2.21: paired t-statistic testing the hypothesis of zero performance difference**

$$t = \frac{(\overline{R_p} - \overline{R_b}) - 0}{\frac{s_a}{\sqrt{T}}}$$

A great deal has been written on the appropriate choice of benchmark, the capacity of active managers to “beat the benchmark”, and their capacity to do so consistently. Even the most appropriately designed benchmark, after costs, is unlikely to represent the mean of the distribution of a fund's periodic returns. Surz (2006) and Sharpe (1991) ascribe the observed failure of the greater portion of funds to beat their benchmarks to the benchmark's off-centre position in the distribution. However, because a good benchmark is known a priori, is investable and measurable, its application to the monitoring of active management performance remains more appropriate than the risk-free rate.

**2.4.2.3 Beyond benchmark adjustments**

Surz (1994) introduced the notion of Portfolio Opportunity Distributions (PODS) as a new innovation in active fund performance assessment. In this technique, the author advocates replacing a time series sampling of fund and benchmark performance with a simulation of the distribution of all possible fund returns over the period of concern. Using the observed returns of individual assets over the period and the rules that govern the construction of the portfolio, a simulator generates a distribution of all the fund returns possible given the manager's constraints and the prevailing market conditions. Both the

realised fund returns and the benchmark returns can be found within this distribution and, by inference, the likelihood of each can be determined with greater statistical validity.

Fund managers who have delivered fund performance significantly above the average of the simulated distribution, Surz argues, have shown strong evidence of investment skill.

This technique can also be used to demonstrate the inappropriateness of the benchmark construction, particularly if the realised returns to this benchmark are well off-centre.

Although this technique was published more than 15 years ago, it has not been widely applied, possibly on account of the sophistication required to simulate the required distributions.

#### ***2.4.2.4 Heteroscedasticity in fund performance***

The aim of this PODS research was to improve the fairness of performance assessment and, in particular, the construction of more appropriate benchmarks. But the methodology used in Surz (1994) also brought into focus the notion of a fund's performance being the realised consequence of the manager's decisions drawn from a distribution of possible alternatives. This in turn draws attention to the deductive nature of the pseudo-t-statistics that are the Sharpe Ratio and Information Ratio, so commonly used in assessing historical fund performance. The purpose of a t-statistic in this context is that it allows us to deduce from a sample of realised performance the likelihood of positive future performance given the sample size and variance of the historical evidence. However, when these fund returns are sampled from a distribution of possible fund returns whose variance varies from one period to the next, the t-statistic loses much of its power.

Bernstein (1998) observed an apparent decrease in the dispersion of fund returns over time and hypothesised that "benchmark fear" and/or increased competition among

professional managers was to blame. But De Silva *et al.* (2001) subsequently argued that the relationship between fund and market dispersion and the increase in fund dispersion subsequent to the performance history observed in Bernstein's 1998 article show that changes in market dispersion over time is the more likely the culprit. De Silva *et al.* (2001) illustrate empirically the varying spread of US fund returns over time and relate the source of this variance to similar heteroscedasticity in stock-specific returns over the same period. The authors find an R squared of 91% between the annual dispersion in active fund performances in the US and the weighted cross-sectional volatility of securities on the US market from 1981 to 2000. They conclude that there must therefore be very little that changes in market efficiency, manager talent or changing trends in asset management (such as a trend towards specialised portfolios or concentrated portfolios) can be contributing to the dispersion in portfolio manager performances. Furthermore, the authors find no evidence of a time trend in the relationship between market dispersion and fund dispersion in the US as was implied by Bernstein's hypothesis of benchmark-fear on the part of fund managers.

Gorman *et al.* (2010), Ankrum and Ding (2002) and De Silva *et al.* (2001), amongst others, point out that both Markowitz's ground-breaking 1952 introduction of a mean-variance paradigm and the Fundamental Law of Active Management<sup>15</sup>, require a time-series estimation of risk. But when active fund managers decide on how to allocate the finite pool of assets under their management among various investments, it is the cross-sectional dispersion of expected returns that is required in order to provide them with a reasonable opportunity for expressing relative preferences. As Gorman *et al.* (2010) points out, if all securities had perfect correlations with each other, there would be no

---

<sup>15</sup> Grinold (1989) and Grinold (1994)

cross-sectional dispersion in their returns and therefore no way for an active manager to achieve excess-of-benchmark performance or incur any active risk. Without cross-sectional dispersion then, the active fund manager would be unable to deliver performance which was in any way distinct from their benchmark or any of their competitors.

Gorman *et al.* (2010) argue that, when an active fund manager makes the forecasts necessary for a portfolio construction decision, they are by implication predicting the cross-sectional variation in security returns. As such, from an ex ante decision-making viewpoint, the authors assert that cross-sectional volatility is more important to the competitive active portfolio manager than the time-series volatility which was central to Modern Portfolio Theory. It follows then that the ex post assessment of the performance of the portfolios under the active manager's control should take this cross-sectional volatility into account and, by implication, the success of the manager in profiting from cross-sectional dispersion.

De Silva *et al.* (2001) argues that the t-statistic for multi-period excess performance assumes a population with a constant variance that necessarily, when applied to a heteroscedastic performance history, results in low significance and therefore little indication of significant fund management skill. The transition to active performance measurement (i.e. subtracting the benchmark's performance) was effectively a correction

for the time-varying first moment of the return distribution<sup>16</sup>. De Silva *et al.* (2001) argues that we should continue to improve on our assessment of performance by correcting for the varying second moment of the distribution as well: heteroscedasticity. The authors acknowledge that the dispersion among fund performances will be smaller in scale than the dispersion among asset returns, because of diversification. However, using the dispersion among benchmark assets as a proxy for the dispersion among fund performances allows an objective correction for heteroscedasticity without being subject to the sample size of similar fund types and acknowledges the likely source of the fund dispersion.

The adjustment recommended in De Silva *et al.* (2001) is akin to that of a weighted OLS regression, where every observation is divided by the weighted cross-sectional standard deviation of the assets in the benchmark. Effectively then, greater weight is given to excess-of-benchmark performance delivered during times of low dispersion where an active manager would be less likely to deliver fund performance substantially different to the benchmark. If this technique were effective in reducing the heteroscedasticity implicit in fund performance, the ability of the standard ex post risk-adjusted performance measurements to detect significant excess performance would be improved.

---

<sup>16</sup> It is noted that the arithmetic of Sharpe (1991) and the argument that the benchmark is essentially a dollar-weighted average and that the unweighted average manager is unlikely to deliver benchmark performance. Benchmark performance is not the most likely estimate of the mean of the distribution of fund performance as both Sharpe (1991) and Surz (2006) argue and to which many articles on the ex post failure of the majority of funds to outperform the benchmark will attest. However, a well-constructed benchmark represents an objective measure of the alternative to attempting to earn excess of benchmark performance by way of the fund in question, namely the investment in a passive portfolio.



Yu and Sharaiha (2007) looks at cross-sectional dispersion as it informs risk budgeting and the relative importance of top-down/bottom-up decision-making process. Yu and Sharaiha (2007) introduces the orthogonal relationship between asset allocation dispersion and stock-selection dispersion, collectively constituting the total return dispersion at any given time. The authors use the visual result of a circle to represent total return dispersion with right-angled triangles representing the segmentation of this dispersion between asset allocation dispersion and stock selection dispersion. This method enables a comparison between these two components of the risk budgeting decision under various market regimes within the common basis of the return-generating opportunities available at the time.

The authors argue that in an active management context, active positions must be taken in a portfolio in such a way that the extent of the position is justified by the size and reliability of the expected excess-of-benchmark returns. As such, Yu and Sharaiha (2007) presents cross-sectional volatility as a method for measuring active management opportunities of markets at any point in time. Yu and Sharaiha (2007) shows that the unweighted cross-sectional dispersion is directly proportional to the returns on a dollar-neutral<sup>17</sup> investment. For this reason, the authors propose the use of cross-sectional volatility in a hedge fund context as a perfect hindsight performance benchmark. This technique allows for the comparison of the dispersion in ex post risk budget (i.e. the areas in which the fund manager spreads or concentrates their portfolio allocations) to the realised cross-sectional opportunity set but requires knowledge of the underlying portfolio compositions. Chapter 5 focuses on fund-level performance measurement and omits analysis that requires knowledge of the composition of the underlying portfolios.

---

<sup>17</sup> Dollar-neutral portfolios have equal quantities of long and short investments

## **Chapter 3 Concentration in the South African equity market and its implied restrictions on the long-only equity fund manager's opportunity set**

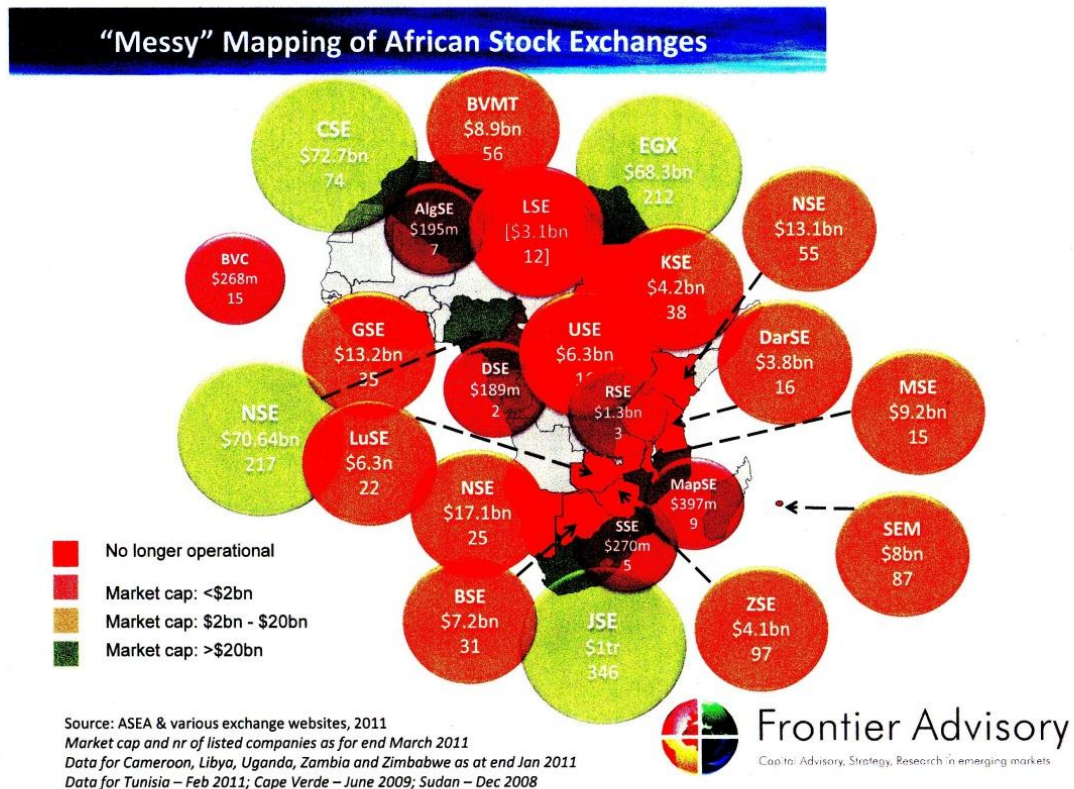
### **3.1 SUMMARY**

South Africa's equity market provides a large (in terms of volume) but concentrated investment environment. Domestic pension funds are restricted from diversifying globally and they are thereby faced with a restricted set of investment opportunities. This chapter describes and quantifies the extent of the concentration on the ALSI historically and at present. This concentration imposes limitations on long-only equity portfolio construction and for the domestic long-only fund manager subject to various active weight limits. The analysis in this chapter shows that the higher the allowable active bet sizes, the less consistently asset managers are able to implement their views and the less symmetric their response to forecasted excess returns can be. Consequently, the less competitive a long-only fund manager can be alongside hedge funds and similarly constrained long-short managers.

### **3.2 CONCENTRATION**

#### **3.2.1 Concentration in South Africa's equity market: present and historic**

The Johannesburg Stock Exchange (JSE) was founded in 1887 and is over 120 years old. It is by far the largest of only 22 African stock markets and ranks 18<sup>th</sup> in capitalisation among the world's stock exchanges: just short of 1,000 billion US Dollars in market capitalisation as of the end of January 2011 (refer Figure 3.1).



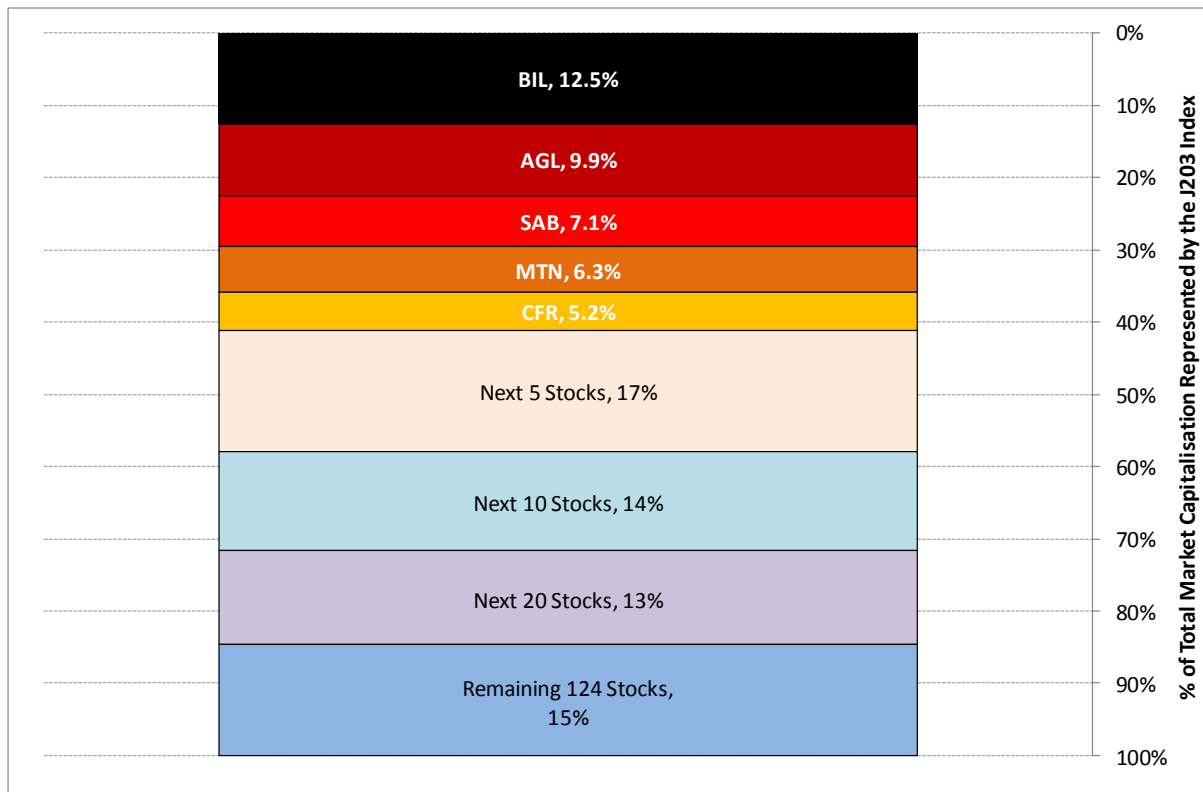
**Figure 3.1: Market capitalisations of African stock exchanges<sup>18</sup>**

Although South Africa's equity exchange is one of the largest among emerging markets, the JSE represents a highly concentrated equity offering. The FTSE/JSE All-Share Index (ALSI) is an index of approximately 164 companies' stocks and represents 99% of the total market capitalisation of all tradable<sup>19</sup> ordinary stocks in South African companies listed on the main board of the JSE<sup>20</sup>.

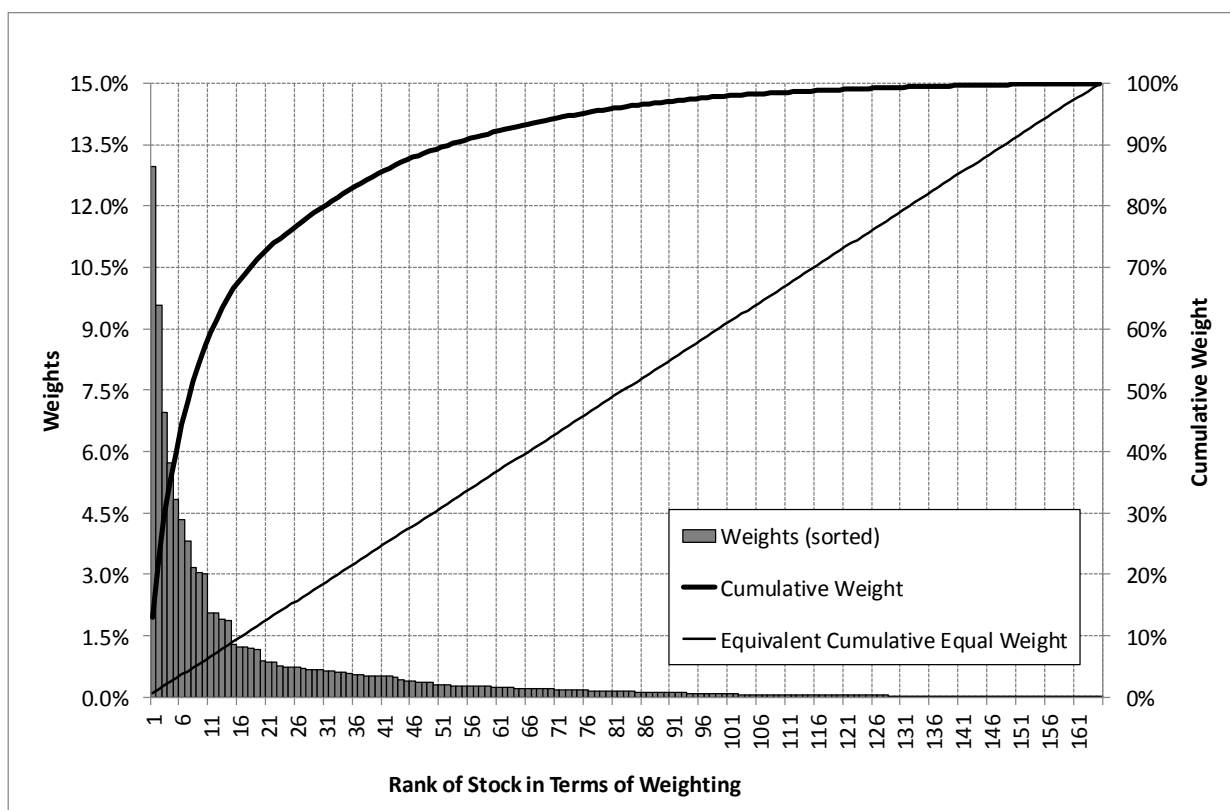
<sup>18</sup> Verachia (2011).

<sup>19</sup> Only "free-float" shares are included in this index.

<sup>20</sup> Ground Rules for the Management of the FTSE/JSE Africa Index Series, July 2009, version 1.8.



**Figure 3.2: The distribution of (market capitalisation) weights on the ALSI (July 2011)**



**Figure 3.3 The cumulative distribution of (market capitalisation) weights on the ALSI (July 2011)**

Figure 3.2 illustrates the concentration of the ALSI by depicting the contribution of various stocks and groups of stocks to the total value of the index as at July 2011. Figure 3.3 depicts the current weight of each individual company in the ALSI as well as the cumulative contribution of each stock to the index's total weight. This cumulative contribution can be compared to the straight dashed line in the figure that represents a hypothetical, equally weighted index comprised of the same equities. An equally weighted index is definitively not concentrated because it is spread evenly across its constituents. The extent to which the curve in Figure 3.3 departs from the straight line represents the extent to which the index is concentrated.

As can be seen in Figure 3.2 and Figure 3.3, the ALSI has more than 20% of its weight in the largest two mining-resources companies. The largest five companies together make up approximately 40% of the index. The seven biggest companies out of the total 164 represent 50% of this index and the largest 15 companies (less than 10% of the number of companies listed) comprise two thirds of this index.

A material portion of the ALSI's market capitalisation is attributable to foreign investment and the dual listing of some of our largest companies on foreign exchanges. For this reason, many domestic investors and investment professionals would argue that the SWIX is more representative of their investment environment. This SWIX weighting technique is an attempt by the JSE/FTSE to represent the South African equity offering excluding the capitalization held by foreigners, which is particularly useful as a benchmark for South African pension fund investment. Since much of this foreign capital is invested in the largest listed companies, the SWIX weighting has a reduced concentration relative to the ALSI and is less dominated by the two largest stocks, Anglo-American (AGL) and BHP

Billiton (BIL). Figure 3.4 and Figure 3.5 illustrate the concentration of the SWIX in the same way that Figure 3.2 and Figure 3.3 illustrated the concentration of the ALSI.

Although the weighting of AGL and BIL are substantially less in the SWIX index, this index nevertheless has only three companies making up 20% of the index. The largest thirteen companies together make up half of the index and the largest 25 companies (15% of the number of companies listed) comprise two thirds of this index. Therefore, whether we consider the South African equity market with or without foreign capital, the main bourse is highly concentrated in large stocks.

In 10 years, the annual volume traded on the JSE Equities markets has increased by a multiple of six (refer to Figure 3.6)<sup>21</sup>. But this increased market activity has not brought a material change to the South African equity market's size ranking in the world nor has it attracted greater diversity in terms of the listings on the main board. In fact, since the existence of the new JSE/FTSE indices, there has been little appreciable improvement in its concentration, as Figure 3.7 will show. Due to the smaller number of companies eligible for inclusion in the ALSI from 1997<sup>22</sup> to 2001 and the relative success of South

---

<sup>21</sup> JSE Market Profile (2011). Available online at <http://www.jse.co.za/Markets/Equity-Market/Equity-Market-Statistics.aspx#Market>.

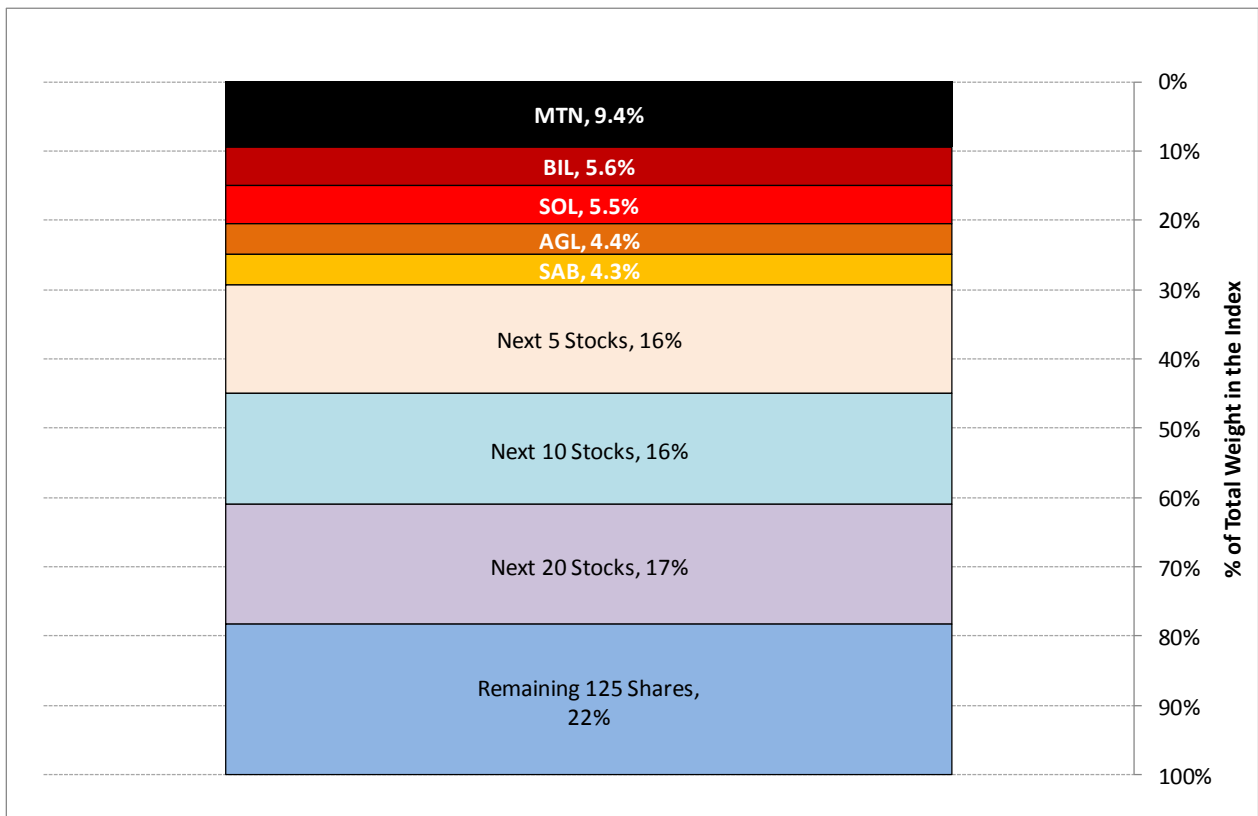
<sup>22</sup> "The JSE Actuaries indices were replaced by the FTSE/JSE Africa Index Series on the 24<sup>th</sup> of June [2002]. FTSE and the JSE provided historic data of the indices for the period July 1995 to December 2001 and the indicative values from the 2nd of January to the 21st of June." - <http://www.jse.co.za/Products/FTSE-JSE/History.aspx>. Although ALSI data is available from an earlier date, the number of constituents in the index halved from 319 in December 1996 to 155 in January 1997 as a result of changes in the rules of index construction. This change in diversity of the index has a material effect on cross-sectional dispersion. For this reason, data prior to January 1997 has been excluded.

Africa's two largest resource companies since 2001, the ALSI has, in fact, become more concentrated over time.

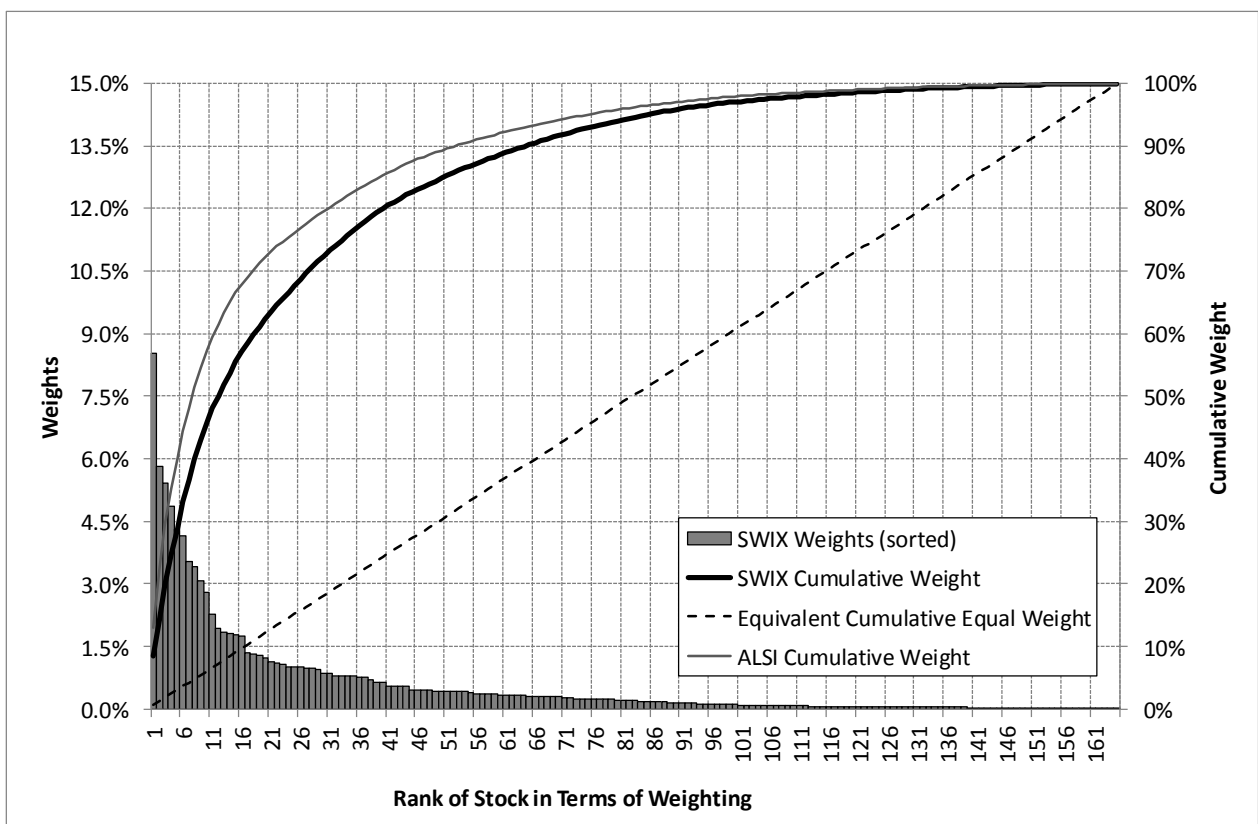
The effective number of stocks measures how many stocks the index would have if it were an equally weighted index, given its actual distribution of weights<sup>23</sup>. On this basis, as shown in Figure 3.7 and Figure 3.8, the ALSI has not had more than an effective 25 stocks since its inception in 2002. Since late 1999, more than half the ALSI's market capitalisation has vested in the top 10 stocks and more than two thirds of the index has been represented by the largest 20 stocks. Considering the fact that the ALSI represents 99% of all tradable equity available for investment in South Africa, this index provides a fairly accurate indication of the limitation placed on investment funds in South Africa.

---

<sup>23</sup> Strongin et al. (2000). Refer to Section 2.3 and Equation 2.13.

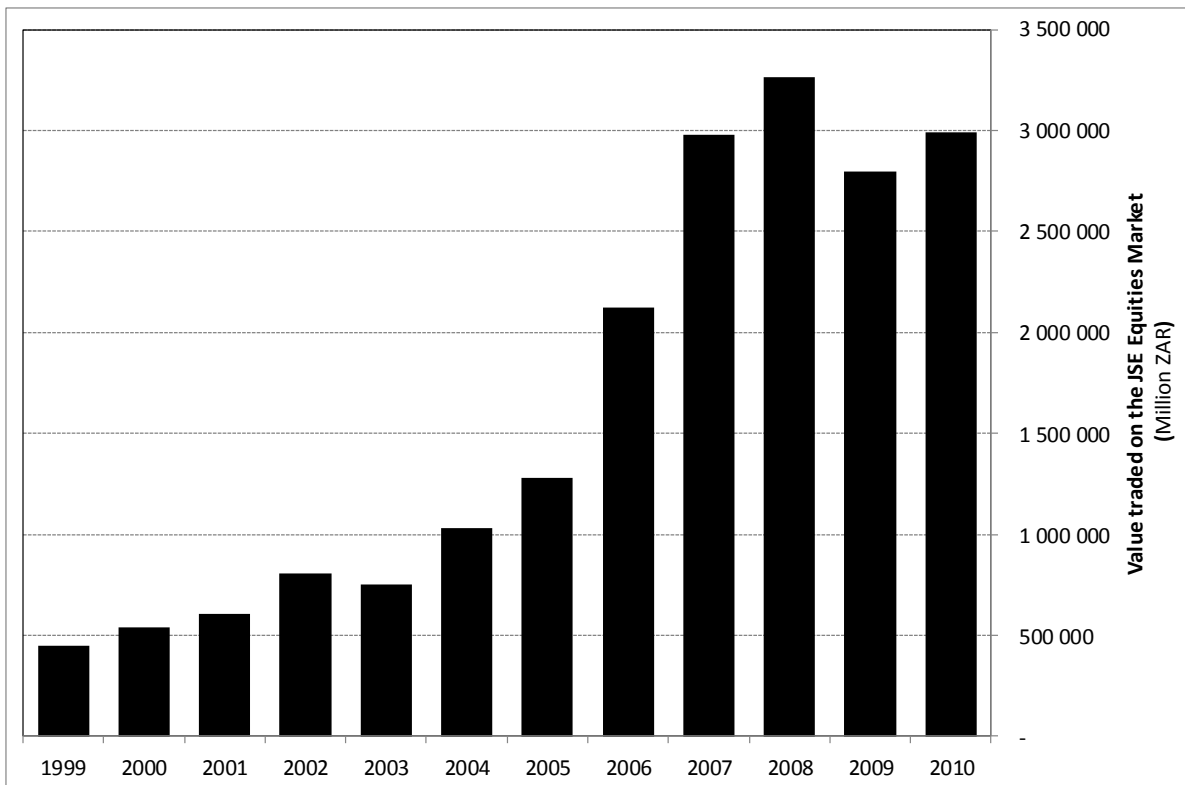


**Figure 3.4 The distribution of (market capitalisation) weights on the SWIX (July 2011)**

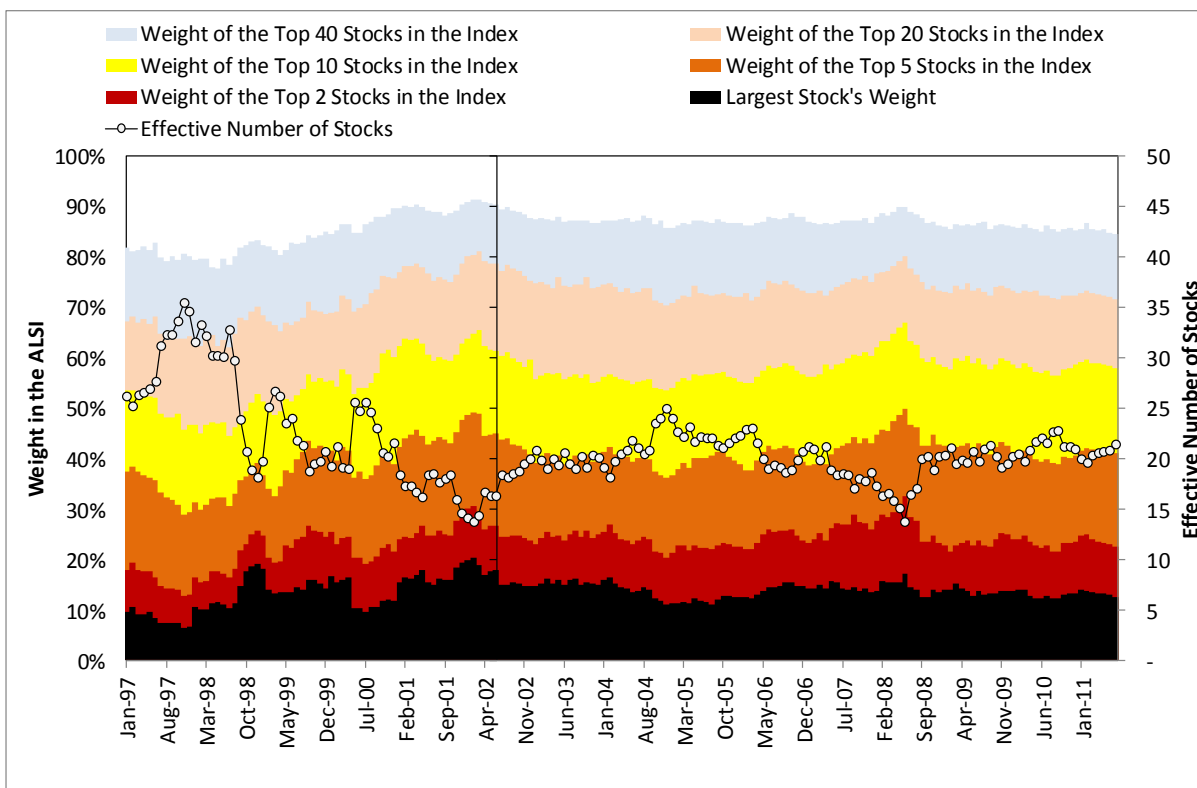


**Figure 3.5 The cumulative distribution of (market capitalisation) weights on the SWIX (July 2011)**

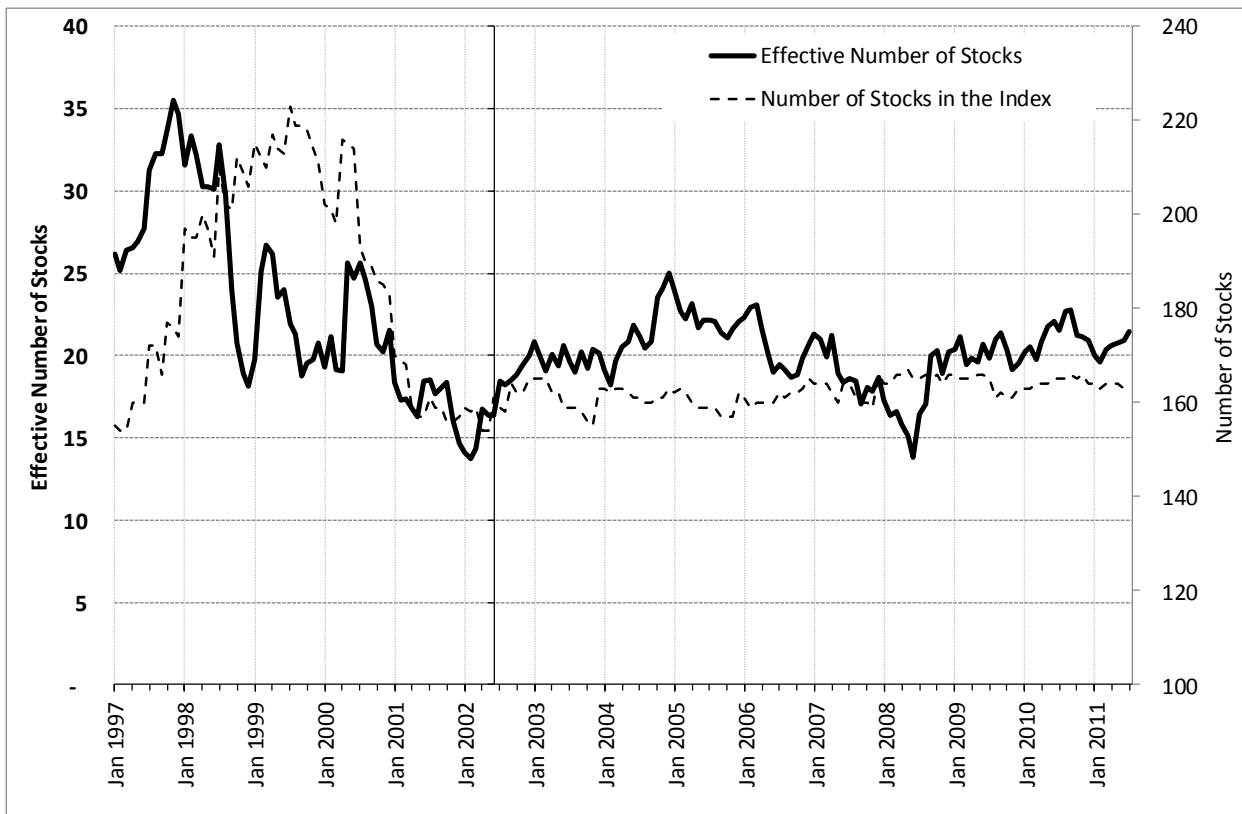




**Figure 3.6: The annual Rand value traded on the JSE**



**Figure 3.7: Concentration of the ALSI over time**



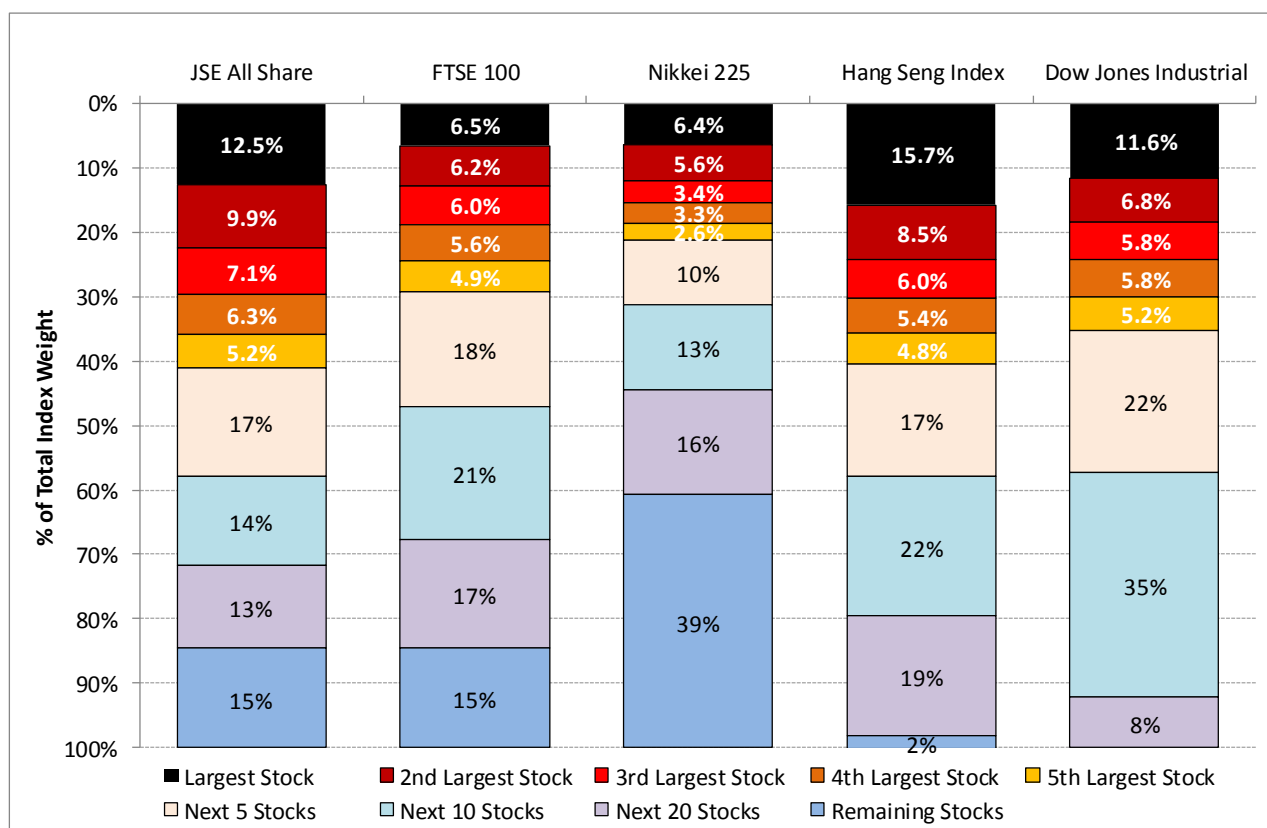
**Figure 3.8: Number and effective number of stocks on the ALSI over time**

### 3.2.2 Concentration in South Africa's equity market relative to developed markets

In a similar way to Figure 3.2, Figure 3.9 compares the concentration of four major global indices to the ALSI. The accompanying Table 3.1 illustrates the concentration of several of these developed market indices and shows that, as an index, the ALSI is not unique in its concentration in a handful of large companies.

The following section of this chapter will show that the concentration of an index or benchmark can materially constrain investment decisions and the portfolio construction process, particularly when each investment is required to be held long. Any investment process that is focussed on long-only investment in a particular, concentrated equity environment will be vulnerable to the constraints of concentrated markets. Global

investors, who segment the management of their funds geographically, mandating long-only investment in each region, will be adversely affected by index concentration in this way. However, when their collective, diverse investment opportunities are considered, the effective concentration in their dollar-weighted investment universe can be greatly reduced by taking a more holistic and less segmented view.



**Figure 3.9: The Distribution of index weights on four major indices and the ALSI**

By contrast, South African investors, who are restricted by exchange controls and investment regulation from full global diversification, have little reprieve from their concentrated investment universe. For this reason, a study of the implications of equity market concentration on fund management opportunities is particularly important in domains where geographical diversification is restricted.

The next section presents an analysis of the loss of opportunity, and implied decrease of the transfer coefficient when long-only mandates are applied in a concentrated market such as the South African equity market. This section begins with an introduction to long-only active management and continues with an illustration of the extent of the decay in the transfer coefficient with increasing active weight limits. The conclusion follows.

**Table 3.1: Index weight of stocks on the ALSI and four other developed market indices**

	ALSI		FTSE 100		Nikkei 225 <sup>24</sup>		Hang Seng		Dow Jones Industrial Average <sup>24</sup>	
<b>Largest Stock</b>	BHP Billiton	12.53%	HSBC	6.52%	Fast Retailing Co Ltd	6.44%	HSBC	15.73%	IBM	11.58%
<b>2nd Largest Stock</b>	Anglo American	9.91%	Vodafone	6.24%	FANUC Corp	5.56%	China Mobile Ltd	8.50%	Chevron	6.75%
<b>3rd Largest Stock</b>	S A Breweries	7.14%	BP	5.98%	Softbank Corp	3.36%	China Construction	6.03%	McDonald's	5.85%
<b>4th Largest Stock</b>	MTN	6.29%	Royal Dutch Shell	5.60%	Kyocera Corp	3.25%	Industrial & Commercial	5.44%	Caterpillar Inc	5.83%
<b>5th Largest Stock</b>	CFR	5.23%	GlaxoSmithKline	4.93%	KDDI Corp	2.58%	CNOOC Ltd	4.78%	3M Co	5.22%
<b>Next 5 Stocks</b>		16.78%		17.81%		10.00%		17.45%		22.12%
<b>Next 10 Stocks</b>		13.74%		20.59%		13.20%		21.63%		34.74%
<b>Next 20 Stocks</b>		13.02%		16.87%		16.36%		18.58%		
<b>Remaining Stocks</b>		15.35%		15.46%		39.25%		2.04%		7.92%
<b>Average Weight</b>		0.61%		0.98%		0.44%		1.00%		3.33%
<b>Median Weight</b>		0.15%		0.35%		0.21%		0.01%		2.64%
<b>No. of Stocks</b>		164		102		225		100		30
<b>Effective number</b>		21		33		61		19		21

---

<sup>24</sup> Nikkei and Dow Jones are price-weighted indices

### 3.3 THE IMPLIED RESTRICTIONS OF MARKET CONCENTRATION ON THE ACTIVE FUND MANAGEMENT OPPORTUNITY SET

#### 3.3.1 The asymmetry of the long-only active portfolio manager's opportunity set

Active portfolio management (as discussed in Section 2.1) is the allocation of fund value amongst available investments, considered relative to a benchmark portfolio. The weight of the portfolio which is invested in any particular stock ( $w_{p,i}$ ) is the proportion of the portfolio's total value invested in this stock. Clearly, the sum of these weights across all investments must add to 100% and the active weights must sum to zero for the investment portfolio to be self-financing. The active fund manager expresses their investment view on the assets in their investment universe by holding an over-, neutral or under-weight position ( $w_{a,i} > 0$ ,  $w_{a,i} = 0$  or  $w_{a,i} < 0$ ) in these assets relative to the manager's assigned benchmark. The extent to which the fund manager can express conviction by increasing a positive active weight is limited only by the restrictions of the fund's rules (legislation<sup>25</sup> and client-specific requirements expressed in a mandate agreement between the fund manager and the client) and the ability to finance this positive active bet with negative active positions in other stocks.

---

<sup>25</sup> Regulation 28, for example, which regulates the investment of South African pension funds, requires that no more than 15% of a pension fund may be invested in a large capitalisation listed stock, and 10% in any single other stock. The proportions apply to the whole fund to which the same regulation restricts the total equity investment to 75%. Considering a maximum equity portion of a pension fund in isolation then, regulation 28 allows a generous 20% (15% of 75%) and 13% (10% of 75%) maximum investment in individual large capitalisation and other listed stocks respectively. The regulatory constraints are not usually binding on a typical, mandate-constrained equity portfolio and the individual mandates which describe the limits of the fund manager's investments are typically more restrictive when it comes to the maximum allowable active or investment weight of the portfolio in any individual stock.

A negative view of the future prospects of a particular stock can be implemented by holding a smaller proportion of such a stock in the portfolio than its proportion in the benchmark. The greater the conviction in this negative bet on this stock, the less of the stock the fund manager will hold. This manager could go so far as to sell the stock short (i.e.  $w_{p,i} < 0$ ) with a view to profiting from its future loss in value, intending to repurchase the stock at a lower price in the future when the short position is cancelled.

However, within a conventional long-only active mandate such as those typically applied to pension funds in South Africa, fund managers may not hold short positions in any asset. For these managers, the most negative expression of their view in a particular asset is to omit it from the fund entirely (i.e.  $w_{p,i} = 0$ ). The smallest investment weight of this stock is therefore limited by zero (i.e.  $w_{p,i} \geq 0$ ) and the most negative active weight possible in a long only fund is therefore the negative of the benchmark weight ( $w_{a,i} \geq -w_{b,i}$ ). In this way, the long-only fund manager has greater scope for expressing positive investment views in each asset than negative views. The conventional long-only fund's prohibition against selling individual assets short therefore has an asymmetrical effect on the active fund manager's opportunity set.

### 3.3.2 The effect of concentration in the presence of active weight limits

It is in this long-only context that the concentration of the benchmark portfolio is particularly relevant. If the benchmark portfolio represented an equally weighted universe of assets, the long-only investment manager would have equal opportunities to express a view in any of the component assets. However, when an asset comprises only a small weight in the benchmark or investment universe, a long-only manager's range of potential active

weights in that stock becomes notably non-symmetrical: the potential positive active weight is uncapped but the size of any negative view is limited by the stock's own weight in the benchmark. The smaller the asset's weight in the benchmark portfolio, the more asymmetric the range of potential active weights will be.

Fund managers are not usually without limits in terms of the risk exposures that they are permitted in their portfolios or the extent of their "active" management. Investment mandates usually stipulate the boundaries of the investment universe (i.e. in what assets may the manager invest), the fund objectives, and the limits to various risks. Among the legislative and mandate requirements are upper limits to the fund's weights and/or active weights in any particular asset. This restriction seeks to avoid concentration of risk in the fund. Regulation 28, for example, puts an upper limit on the weight of a pension fund in particular assets to ensure that the fund as a whole is diversified.

In an active management context, some mandates restrict the active weights of the fund across all assets to limit both the size of active risk and to ensure diversification of the portfolio manager's activities. An index fund for example would require active weights of zero throughout. A so-called "enhanced index" manager would be subject to very small upper limits on active weights, as these manager's provide investors with very low active risk, typically at a reduced management fee, with fund performances that are not expected to vary substantially from the benchmark. Although enhanced index funds are active funds, exercising the views of active fund managers, they do so with very modest and broadly diversified active weights and offer an alternative to a strictly passive investment in an index. An "aggressive" active manager, by contrast, would be expected to take much larger active positions when exercising their investment views and are typically paid larger



management fees for this activity. The performance of these aggressive mandates is expected to vary substantially from the benchmark. These aggressive managers are not necessarily expected to consistently succeed to deliver superior fund returns relative to their benchmarks at all times but they would be expected to consistently take substantial active positions with the fund's assets.

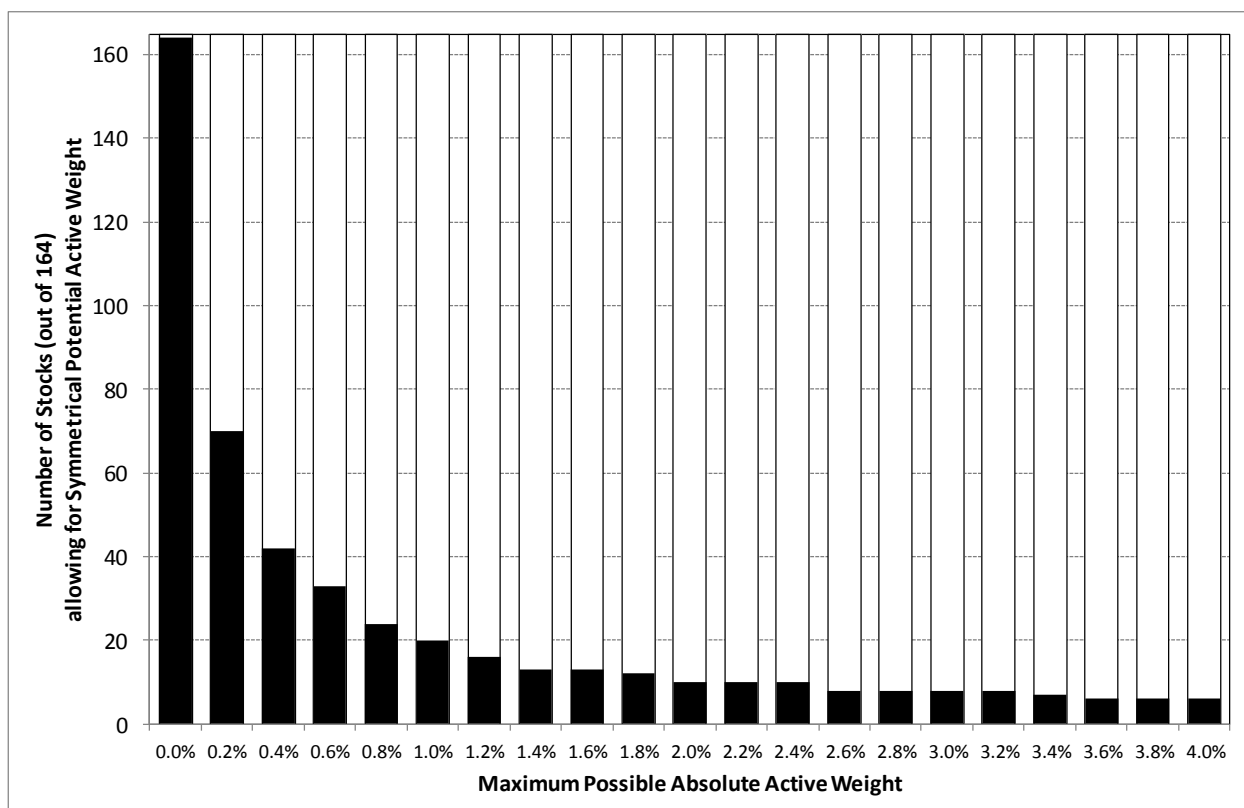
Figure 3.10 illustrates the implications of various sizes of such active weight limits on the potential activity of a fund manager in the concentrated equity market in South Africa. The horizontal axis represents the relaxation from left to right of maximum allowable active weights in any stock in absolute terms (i.e. the restriction applies to both positive and negative active weights). For example, the extreme limit of 0% would represent a perfect ALSI tracking fund whereas the 3% limit represents a mandate with more scope for aggressive active management because it allows for an active weight in any stock of anywhere between -3% and 3%. The vertical axis of Figure 3.10 represents the number of stocks available to the fund manager, under each of these restrictions, in which the manager has a symmetrical range of opportunity to express a negative or positive active view.

For example, at a very conservative active weight limit of 20 basis points<sup>26</sup> (bp), the thus constrained active manager can potentially express active views anywhere between -20bp and 20bp on only 70 out of the 164 stocks in the ALSI (i.e.  $-.20\% \geq w_{a,i} \geq .20\%$ ). This implies that the manager can give symmetric consideration of the opportunities for extra

---

<sup>26</sup> 20bp = 0.20%

profit from only 70 of the 164 available stocks. The maximum of 20bp positive active position is permissible in each of the 164 stocks but a negative active position of 20bp is only possible in the largest 70 stocks. The most extreme negative active position possible in the remaining 94 smaller stocks is limited by these stocks' own weights in the benchmark, which are all less than 20bp. The fund manager must therefore consider any potential opportunity for excess returns in these 94 smaller stocks in an asymmetrical way because there is greater potential for expressing positive active views in these stocks than negative views. All things being equal, there would therefore be less point in the manager paying attention to "sell" signals in these stocks than to "buy" signals as the manager is less able to orientate the fund to take advantage of opportunities for profiting from negative forecasts in these stocks.



**Figure 3.10: Maximum number of stocks available at each level of active weight**

Under a less restrictive limitation, for example a maximum allowable active weight of 2%, the manager can only consider 10 stocks in which the full range of potential active weights are possible. That is, ironically, while the 2% limit is far less restrictive, in a concentrated market it effectively only allows the more aggressive manager the full range of opportunity to express their more aggressive range of views (from -2% to 2%) in the largest 10 out of the 164 stocks in their investment universe. Thus with greater relaxation of active weight limits comes greater asymmetry in the range of opportunities available to the fund manager.

Notice from Figure 3.10 how quickly the opportunity set of stocks reduces. An index (passive) fund (0% active weight limit) invests across all 164 stocks in the same way with zero active weights. At a maximum allowable active weight of 20bp, the number of stocks in which the full range of opportunities is available more than halves. There is a further 40% reduction in the opportunity set with a change in active weight limit from a tiny 20bp to a very slightly larger 40bp. At a 2% maximum active weight, the opportunity set is down to only ten stocks. This gives us an indication of the small sample of stocks in which long-only, active managers can meaningfully express their views, both negative and positive, across the full allowable range of active weights. Furthermore, it shows how very limited the field is for aggressive active managers in this context.

In a similar vein to Figure 3.10, Table 3.2 describes the limitations to the opportunities of a long-only fund benchmarked against the ALSI as a consequence of active weight limits. For example, a manager who is constrained to a maximum 0.02% active weight limit in any stock in the index, has a total range of 32.8% (0.2% times 164) of positive active positions (refer column 2 Table 3.2). An unconstrained manager could finance these with

symmetrical short positions, thereby creating a total opportunity set of 65.6% (twice 32.8%) possible active weights (refer column 5 Table 3.2). By contrast, a constrained long-only fund manager, who is unable to sell a stock short, cannot take a negative active position that is more negative than each stock's own weight in the benchmark. Such a manager would be able to generate a maximum of 0.2% underweight in each of only 70 of the larger stocks. The maximum possible negative position in the remaining 94 stocks is limited by their individual benchmark weights, which in this case sum to 6.3%. As such, the long-only manager with a 0.2% active weight limit would have a range of only 20.3% (0.2% time 70 plus 6.3%) in total negative active weights (refer column 3 Table 3.2). Therefore, the total opportunity range for a long-only manager with a 0.2% maximum active weight limit is 53.1% (32.8% plus 20.3%) possible active weights (refer column 4). This implies that such a manager has only 81% (53.1% of 66%) of the opportunities that an unconstrained manager with the same active weight limit has (last column of Table 3.2). The last column of Table 3.2 records the ratio of opportunities for the long-only manager relative to that of the unconstrained manager who is subject to the same active weight limit. I refer to this metric, represented by the last column of Table 3.2, as the implied transfer coefficient. The derivation of this metric follows in Section 3.3.2.1.

Figure 3.11 summarises the same evidence presented in Table 3.2 by charting the decrease in implied transfer coefficient with increasing active weight limits. From the active weight range of a pure tracker to a very conservative enhanced index fund, the loss of opportunity decreases exponentially. Note the rapid decay in implied transfer coefficient within the first, relatively conservative active weight limits. At a fairly moderate active management limit of 1% for example, the long-only investment in the ALSI is already hamstrung to the order of 35% relative to its unconstrained counterparts. At an

active weight limit of 15% (larger than the largest stock in the ALSI), the decay in the implied transfer coefficient reaches its minimum of 0.52 - roughly half the opportunities lost<sup>27</sup>.

**Table 3.2: Restricted opportunities at various active weight limits for the long-only investor in the ALSI**

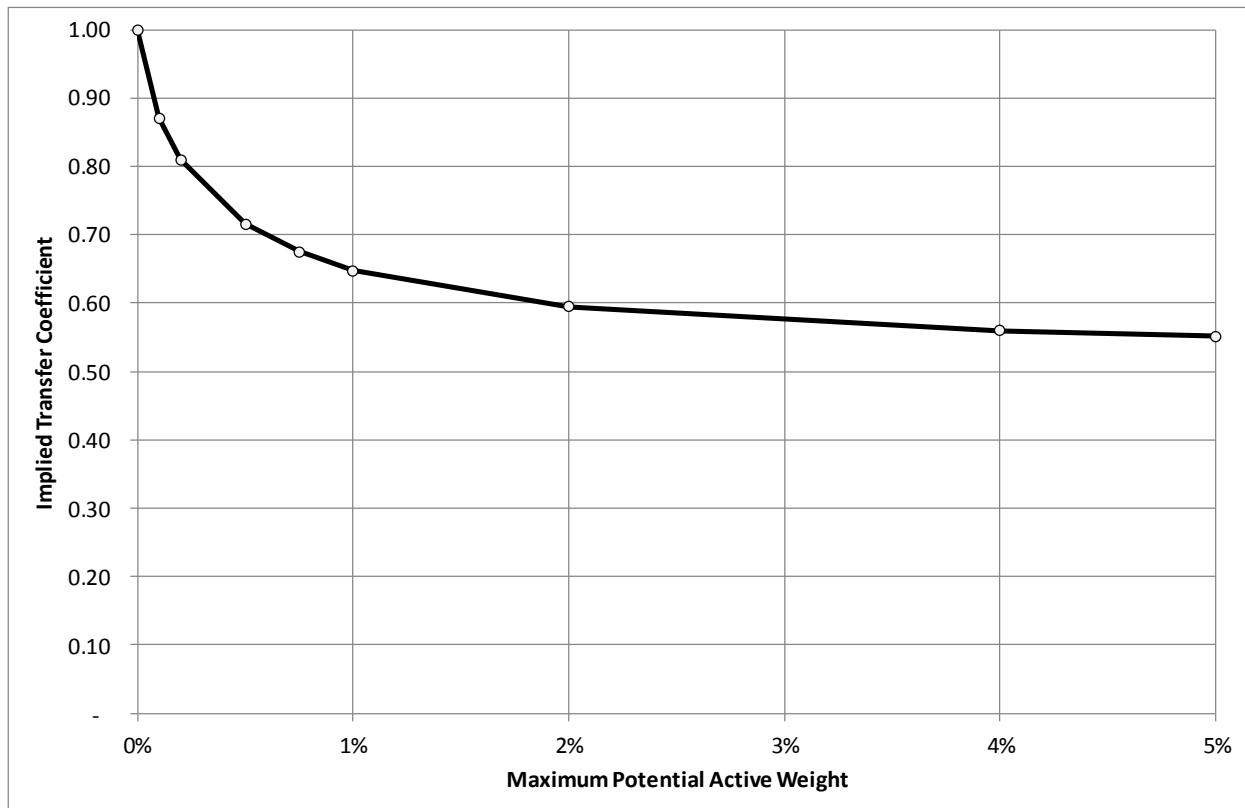
Maximum active weight limit ( $w_a^{max}$ )	Sum of maximum possible positive active weights	Sum of maximum possible negative active weights: long only	Maximum possible active weight range: long only	Maximum possible active weight range: unconstrained	Implied Transfer Coefficient
0.00%	0%	0%	0%	0%	<b>1.00</b>
0.10%	16%	12%	29%	33%	0.87
0.20%	33%	20%	53%	66%	0.81
0.50%	82%	35%	117%	164%	0.72
0.75%	123%	43%	166%	246%	0.67
1.00%	164%	48%	212%	328%	0.65
2.00%	328%	62%	390%	656%	0.59
4.00%	656%	78%	734%	1312%	0.56

Clearly, with increased maximum active weight limits, come increased opportunities for active management and increased expectation of risk taking relative to a passive investment. However, Figure 3.11 shows that with increased active weight limits, the long-only manager's opportunity set becomes increasingly restricted and asymmetric

---

<sup>27</sup> The axis in Figure 3.11 is only shown from 0 to a 5% active weight limit.

relative to a long-short manager thereby allowing a long-short manager an increasing competitive advantage in a more aggressive active management environment.



**Figure 3.11: Decreasing implied transfer coefficients with increasing active weight limits**

### 3.3.3 The implied transfer coefficient

The transfer coefficient was introduced by Clarke *et al.* (2002) (refer Section 2.1.1). It is defined as the cross-sectional correlation of the risk-adjusted forecasts across assets and the risk-adjusted active portfolio weights in the same assets. Calculating the transfer coefficient enables an assessment of the inefficiencies introduced by the mandated restrictions of the portfolio and can assist fund sponsors and mandate authors in their understanding of the impact of constraints on the portfolio's performance. In order to calculate the transfer coefficient of any fund at any particular point in time, one would need to know the forecasts and model estimates of residual risk for every asset in the

investment universe at the time. This is the kind of information to which only the portfolio managers usually have access. Furthermore, the effect of mandated constraints on portfolio weights will vary to some extent with the forecast returns. For example, in a long-only fund, a negative view of larger assets is more easily implemented than a positive view on large assets because the negative active weight in larger assets frees up financing for a variety of positive positions. By contrast, a positive active weight in large assets requires many small negative active weights to finance such a position, irrespective of whether those positions are justified by the forecasts, thus reducing the transfer coefficient of the resulting portfolio. The larger the active weights in the portfolio (which are usually as a consequence of a larger risk budget), the greater the asymmetry of the investment opportunity set and the smaller the transfer coefficient of the portfolio.

While the transfer coefficient provides an appropriate metric for the fund manager, the fund sponsor requires a method of measurement with less onerous information requirements and one that has validity beyond the forecast and forecast period. This section proposes a simple, forecast-independent metric for the inefficiency in implementation implied by portfolio weight constraints: the implied transfer coefficient. The implied transfer coefficient is a measure of the active fund manager's ability to implement an investment view as implied by the benchmark composition and the security-level constraints. Unlike the transfer coefficient, this metric requires only the weights of the benchmark assets and the investment weight constraints. Although the implied transfer coefficient is no substitute for an accurate measure of transfer coefficient, it provides fund sponsors with a quick, easy and intuitive calculation that can assist in the mandate setting process without requiring information specific to the investment view of the manager.

The implied transfer coefficient, as described by Equation 3.1, compares the sum of the range of active positions on each asset in a long-only fund (the numerator) to that of an unconstrained fund (the denominator) in the same way as was presented by Table 3.2 and the related discussion.

### Equation 3.1: Definition of Implied Transfer Coefficient (ITC)

$$\text{ITC} = \frac{nw_a^{\max} + \mathbf{1}^T \cdot \min[\mathbf{w}_b, w_a^{\max}]}{2nw_a^{\max}}$$

where

$w_a^{\max}$  is the maximum permitted active weight in absolute terms as prescribed by the mandate limits;

$\min[\mathbf{w}_b, w_a^{\max}]$  is the  $n \times 1$  matrix of the smaller of each asset's weight in the benchmark or the maximum investment weight permitted;

$\mathbf{1}^T$  is a  $1 \times n$  vector of ones and  $n$  is the number of assets in the investment universe.

The denominator of Equation 3.1 ( $2nw_a^{\max}$ ) represents the full permissible active weight range of the unconstrained fund manager in each of  $n$  assets in the investment universe: from  $w_a^{\max}$  to  $-w_a^{\max}$ . The long-only manager has a similar range in terms of the positive active weights in each of the assets ( $nw_a^{\max}$ ). However, in terms of the possible negative active positions, the long-only manager can only hold the maximum negative active positions ( $-w_a^{\max}$ ) without having to sell the asset short if the asset's benchmark weight is larger than the active weight limit (i.e. for assets such that  $w_{b,i} \geq w_a^{\max}$ ). For smaller assets (i.e. assets such that  $w_{b,i} < w_a^{\max}$ ) the maximum possible negative active positions possible without incurring short positions is  $-w_{b,i}$ .



Figure 3.12 illustrates the implied transfer coefficient using an ALSI benchmark. The asymmetry and loss of opportunity as measured by the implied transfer coefficient is evident in long-only funds at very conservative active weight limits and worsens with greater permissible active weights. At a maximum of 1% active weight, for example, approximately one third of the opportunity set is unavailable to the long-only active manager who has an implied transfer coefficient of 0.65 (as shown earlier in Table 3.2 and Figure 3.11).

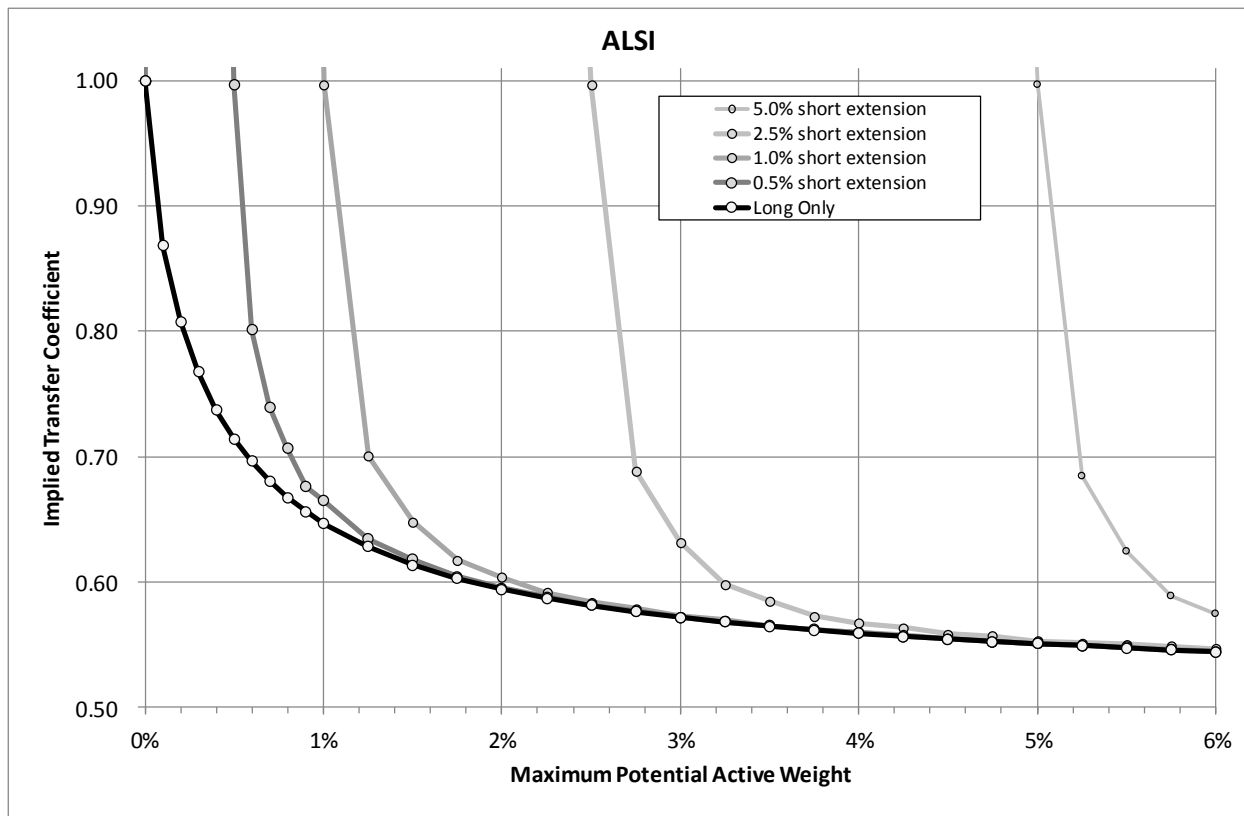
### 3.3.4 Short extension funds

Using this simple implied transfer coefficient statistic, which is independent of investment view, market conditions and all risk and portfolio self-financing considerations, this section attempts to quantify the benefits of a very small short extension to the long-only fund manager (refer to Chapter 1 Section 2.2 for a discussion on short extension funds). To this end, the implied transfer coefficient in Equation 3.2 is generalised to allow for greater range in the maximum negative active weights as would be the case in a short extension fund.

#### Equation 3.2: Generalised ITC allowing for short extension (X)

$$ITC = \frac{nw_a^{max} + \mathbf{1}^T \cdot \min[w_{b,i} + X \cdot \mathbf{1}, w_a^{max} \cdot \mathbf{1}]}{2nw_a^{max}}$$

where X is the maximum permissible sum of short positions in the fund e.g. a so-called “130/30” fund has an X of 30% and a long-only fund has an X of zero.



**Figure 3.12: Implied Transfer Coefficient for an ALSI<sup>28</sup> equity fund**

The denominator is unchanged as the comparison to the unconstrained fund remains.

The number of assets for which the maximum active position is permissible, increases because of the short extension ( $X$ ): all the assets for which  $w_{b,i} + X \geq w_a^{max}$  can be held at the maximum negative active position ( $-w_a^{max}$ ). Likewise, there are fewer assets for which the full range of active weights cannot be achieved: assets for which  $w_{b,i} + X < w_a^{max}$  can only have maximum negative active positions up to the benchmark weight plus the short extension i.e.  $-(w_{b,i} + X)$ .

Returning to Figure 3.12, the implied transfer coefficients for four very modest short extension products on the ALSI are depicted alongside the long-only fund to illustrate the

<sup>28</sup> Using the ALSI constituents as at end of July 2011.

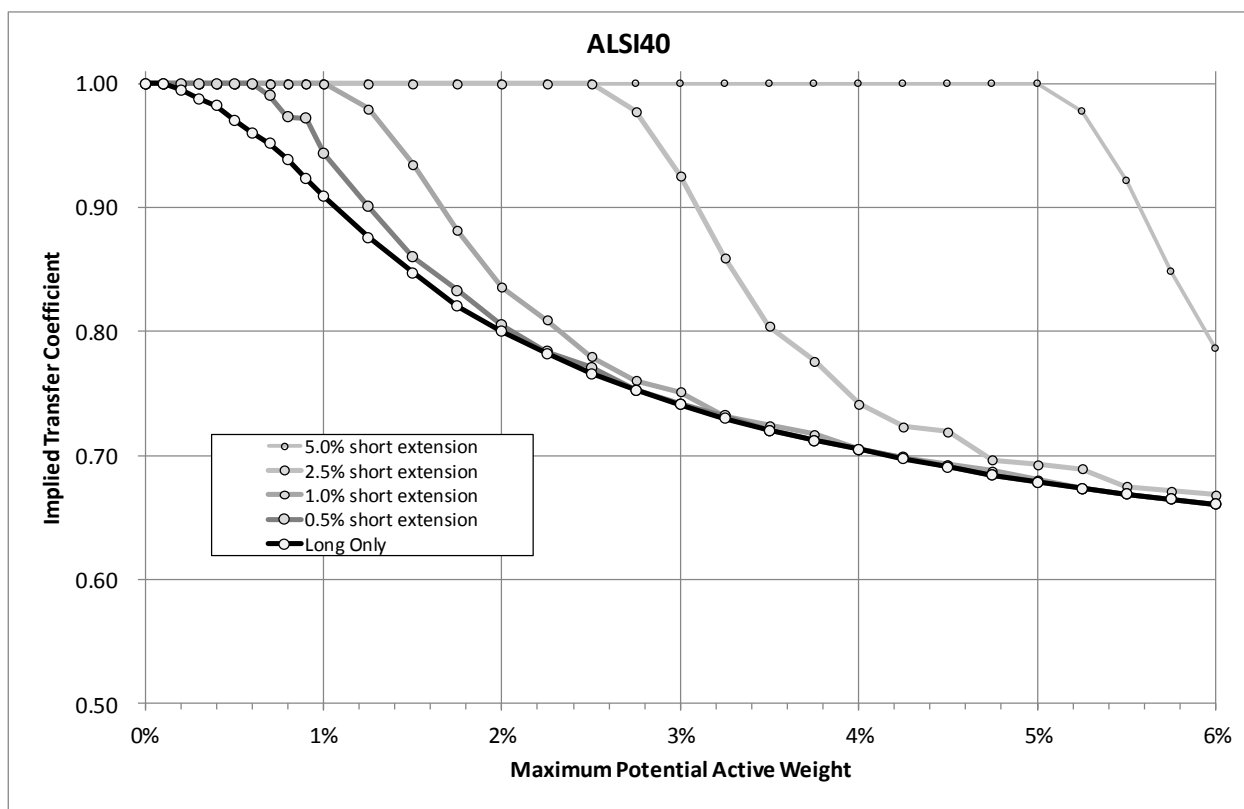
improvement in the opportunity set for net long active managers for whom the short selling restriction is very slightly relaxed. For example, at a 3% active weight limit, the long-only manager is at a 0.57 implied transfer coefficient, a substantial disadvantage compared to their unconstrained peers. However, a mere 2.5% short extension will improve this manager's implied transfer coefficient to 0.63 and a 5% short extension will essentially put the active manager on equal footing with their unconstrained counterparts in terms of their opportunity set.

Referring to the Generalised Fundamental Law of Active Management (refer Section 2.1.2 Equation 2.11), the Information Ratio (Equation 2.20) is a function of the Transfer Coefficient (TC). Therefore, if two managers both have the same investment opportunities from which to choose and the same success at predicting returns, the one with the higher Information Ratio will, by definition, be the one who best reflects their expected risk-adjusted returns in their portfolio i.e. the one with the higher transfer coefficient. Likewise, the asymmetry of opportunities represented by the implied transfer coefficient for long-only managers relative to their unconstrained counterparts is likely to translate into poorer risk-adjusted performance as well, although not in a directly proportional way. Investors and their managers would therefore do well to review their active weight limits in the light of the concentration of their investment universe and benchmarks.

### **3.3.5 A comparison of South African equity indices**

The consequences of limits on active weights are less for fund managers with a more specialised universe and a larger weight per asset. Figure 3.13 depicts the same implied

transfer coefficients and weight limits as Figure 3.12 but based on a benchmark comprised of only the largest forty stocks: the ALSI 40. The loss of efficiency and asymmetry in investment opportunities is less for a fund with an ALSI 40 benchmark (median weight 1.06%) compared to a more diverse ALSI benchmark (median weight 0.15%). Consequently, the need for short extensions to improve the efficiency of these portfolios is less.



**Figure 3.13: Implied Transfer Coefficient for an ALSI 40 equity fund**

Table 3.3 is a summary of some well-known FTSE/JSE indices and comprises the implied transfer coefficients for each of these indices under long-only constraints. Overall, the smaller indices with fewer constituents and larger overall weightings per constituent present the long only active manager with a more evenly distributed opportunity set for active allocation.

This section has provided some empirical indication of the material restrictions under which long-only, benchmarked fund managers are placed, particularly when the benchmarks have high concentration in a few members and a large number of smaller weightings in others. In this context and using this very simple metric, the benefits of small short extensions to the investment range of these same active fund managers have been presented. The evidence suggests that small short extensions (less than 5% overall) may substantially improve the opportunity set for these managers.

Table 3.3: Implied transfer coefficient of long-only funds with various FTSE/JSE benchmarks

$w_a^{max}$	ALSI	SWIX	CAPI	ALSI 40	SWIX 40	CAPI 40	RAFI 40	MID CAP	SMALL CAP	RESI	INDI 25	FINI 15	FINI 30
(%)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.10	0.87	0.91	0.87	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.20	0.81	0.84	0.81	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.30	0.77	0.81	0.77	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00
0.40	0.74	0.78	0.74	0.98	0.99	0.98	0.99	0.99	0.99	0.98	1.00	1.00	1.00
0.50	0.72	0.75	0.72	0.97	0.99	0.98	0.99	0.98	0.99	0.98	1.00	1.00	1.00
0.60	0.70	0.73	0.70	0.96	0.98	0.97	0.99	0.98	0.98	0.97	1.00	1.00	0.99
0.70	0.68	0.72	0.69	0.95	0.97	0.96	0.98	0.97	0.97	0.97	0.99	1.00	0.99
0.80	0.67	0.70	0.67	0.94	0.96	0.95	0.98	0.96	0.96	0.97	0.99	1.00	0.99
0.90	0.66	0.69	0.66	0.92	0.95	0.94	0.97	0.96	0.95	0.97	0.98	1.00	0.98
1.00	0.65	0.68	0.65	0.91	0.94	0.92	0.96	0.95	0.94	0.96	0.97	1.00	0.98
1.25	0.63	0.66	0.63	0.88	0.92	0.89	0.93	0.92	0.92	0.95	0.95	1.00	0.97
1.50	0.61	0.64	0.62	0.85	0.90	0.86	0.90	0.89	0.89	0.93	0.93	1.00	0.94
1.75	0.60	0.63	0.61	0.82	0.87	0.84	0.87	0.87	0.87	0.92	0.91	1.00	0.91
2.00	0.59	0.61	0.60	0.80	0.85	0.81	0.84	0.84	0.85	0.91	0.88	0.99	0.89
2.50	0.58	0.60	0.58	0.77	0.82	0.78	0.80	0.80	0.80	0.90	0.84	0.97	0.85
3.00	0.57	0.59	0.57	0.74	0.78	0.75	0.77	0.76	0.76	0.89	0.81	0.96	0.81
4.00	0.56	0.57	0.56	0.70	0.74	0.72	0.73	0.70	0.70	0.87	0.76	0.94	0.76

### 3.4 CONCLUSIONS

Active fund managers can only express their views in an environment where their conviction and level of risk taking is commensurate with their constraints. The higher the allowable active bet sizes, the less competitive a long-only fund manager can be alongside hedge funds and similarly constrained long-short managers. A concentrated benchmark/investment environment such as the ALSI where only a few of the stocks comprise most of the total investment weight exacerbates this competitive disadvantage.

The more constrained the investment environment, both with regard to mandated constraints and the concentration of South Africa's equity market, the less consistently asset managers are able to implement their views and the less symmetry there is in their range of potential responses to forecasted excess return. Short sale restrictions, in particular, are intended to avoid incurring a liability on the portfolio's behalf. However, the impact of short sale restrictions combined with mandated constraints on active weights in a concentrated market serve, not to limit risk levels, but to artificially concentrate the level of active investment activity in a handful of listed companies.

The disadvantage to active management within more aggressive active weight allowances, speaks to the success of low active risk, enhanced-index type strategies in the South African market. In a long-only, concentrated environment, low risk active strategies provide investors with the best "bang for their buck" because long-only fund managers have the opportunity to act more fully on their active views across the full cross-section of available equities at these low active weight limits. By contrast, to compel or encourage long-only managers into a more aggressive active space in a concentrated investment

environment is, ironically, only to constrain them further in their abilities to express their best active view.

Chapter 4 and Chapter 5 explore the influence and importance of the variation in the cross-sectional dispersion represented by the investment universe in the portfolio construction process, both ex post and ex ante. Thereafter, Chapter 6 picks up where this chapter has left off by exploring active weights and their constraints under various market conditions. Finally, Chapter 7 measures the distribution of various active weight related metrics to illustrate the detrimental effects of long-only constraints under various conditions.



## **Chapter 4 : Varying cross-sectional volatility in the South African equity market and the implications for the management of fund managers.**

### **4.1 SUMMARY**

Modern portfolio theory is founded on an understanding of longitudinal volatility but it is the *cross-sectional* dispersion among investment returns that provide active portfolio managers with their competitive investment opportunities. The varying cross-sectional volatility in the South African equity market provides varying opportunity sets for active managers: the higher the cross-sectional volatility, the greater the opportunity for active risk taking, all other things being equal. This chapter argues that cross-sectional volatility must be considered hand-in-hand with risk limits and active risk targets when investment mandates are set and when mandated risk compliance is monitored.

### **4.2 INTRODUCTION**

This chapter begins with an empirical and historical analysis of the changes in cross-sectional volatility in the South African equity market, showing the extent to which cross-sectional variation has changed over time. The following section explores the impact of varying cross-sectional dispersion on mandate and fund objective setting, particularly the effect on actively managed funds when active risk expectations are held constant.

Furthermore, this section looks at the on going monitoring and management of active funds' mandate compliance with respect to active risk as a result of changes in cross-sectional dispersion of securities.

### 4.3 HISTORICAL CROSS-SECTIONAL DISPERSION ON THE ALSI

Following the methodology of Ankrum and Ding (2002), the weighted cross-sectional volatility for any investment period is defined here and calculated as follows:

#### Equation 4.1: Realised cross sectional variance

$$s_{cs,t}^2 = \sum_i^N w_{i,t-1} (R_{i,t} - R_{b,t})^2$$

where:

$s_{cs,t}^2$  is the weighted realised cross-sectional (cs) variation of a particular benchmark or index over a particular investment period;

$w_{i,t-1}$  is the weight (typically market capitalisation) of each stock,  $i$ , at time,  $t - 1$ , in the benchmark;

$R_{i,t}$  is the total return<sup>29</sup> for each stock,  $i$ , from time  $t - 1$  to time  $t$ ; and

$R_{b,t}$  is the corresponding benchmark/index return which is the weighted average return across stocks over the same period (i.e.  $R_{b,t} = \sum_i^N w_{i,t-1} R_{i,t}$ ).

---

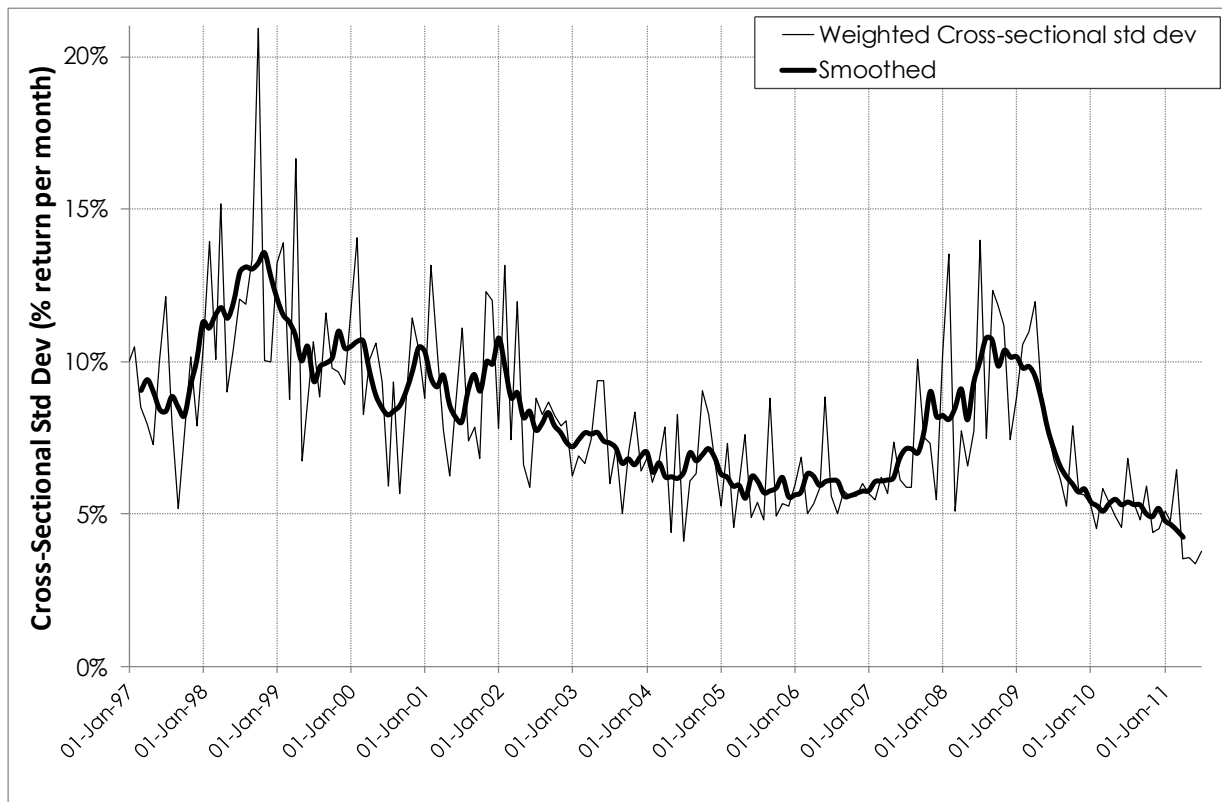
<sup>29</sup> The total return was calculated by including all dividends (including scrip) on the applicable ex dividend date into the return calculation. Total return therefore represents both capital gain (growth in the share price) and dividend yield.

Figure 4.1 depicts the weighted monthly<sup>30</sup> cross-sectional standard deviation (the square root of Equation 4.1) over time of the ALSI (the bold line is a 6-month moving average of the same). Note the extent of the change in cross-sectional standard deviation over time: from low levels in the late-90's<sup>31</sup>, increasing in just over three years to peak in the recovery just after the 1998 market crash (the maximum to date of 21% occurred in October 1998). The slow decline in cross-sectional volatility from 1998 seemed to have hit its lowest point in our market between 2004 and 2006 (the minimum of 4% occurred in July 2004). From 2007 the cross-sectional volatility increased again only to fall sharply to a new low in 2011 (4% in July 2011). Although the history presented here is shorter than that presented by Bernstein (1998) and De Silva *et al.* (2001), we see similarly low levels of dispersion on the ALSI in the mid to late 1990's picking up at a similar time to the US evidence presented in De Silva *et al.* (2001). However, 1999 and 2000 saw unprecedented high levels of cross-sectional dispersion in the US where, in South Africa, ours had already declined substantially. Evidence presented by Sapra (2008), over the time of the 2008 financial crisis, showed that the US market appeared to be returning to these high levels again in much the same way the South African market has in 2008. The most recent year has seen what appears to be a very rapid decline in cross-sectional dispersion of opportunities on the ALSI.

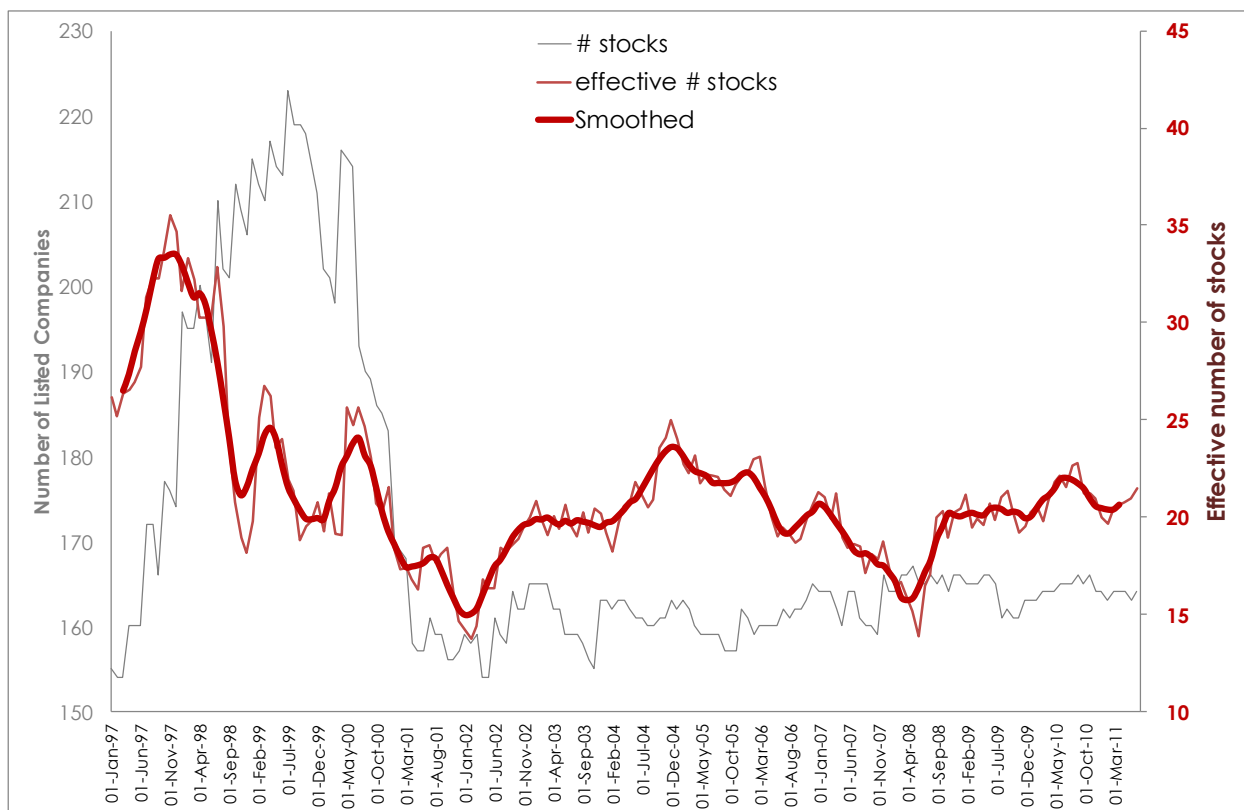
---

<sup>30</sup> It's typical in a portfolio management context to use monthly performance statistics as a high frequency performance monitoring tool towards quarterly and annual performance reports to the client. It is not common practice to report on fund performance at a higher frequency.

<sup>31</sup> Although ALSI data is available from an earlier date, the number of constituents in the index halved from 319 in December 1996 to 155 in January 1997 as a result of changes in the rules of index construction. This change in diversity of the index has a material effect on cross-sectional dispersion. For this reason, data prior to January 1997 has been excluded.



**Figure 4.1: Historical cross-sectional standard deviation on the ALSI**



**Figure 4.2: Effective and actual number of stocks on the ALSI over time**

In their study of developed market cross-sectional correlation, Ankrum and Ding (2002) suggested that one of the reasons for the increase in cross-sectional correlation that they had observed was that listings may be taking place earlier in the life cycle of companies. Presumably they were pointing to a greater diversity in listed equities as well as a lower concentration in the relative weighting of such equities. By way of comparison, Figure 4.2 (a repetition of the information in Figure 3.8) shows the number of stocks and the effective number of stocks<sup>32</sup> on the ALSI since January 1997.

The ALSI has seen a marked decline in both the number of stocks and an increase in concentration from 1998 to 2001 and that could certainly have contributed to the declining cross-sectional opportunities evident in the early part of this century shown in Figure 4.1. However, after 2001 there doesn't seem to be any particular relationship between the concentration of the index and the variation in its stocks' performances. The ALSI (free float adjustments considered) currently stands at approximately 20 effective stocks.

#### **4.3.1 Sectoral evidence**

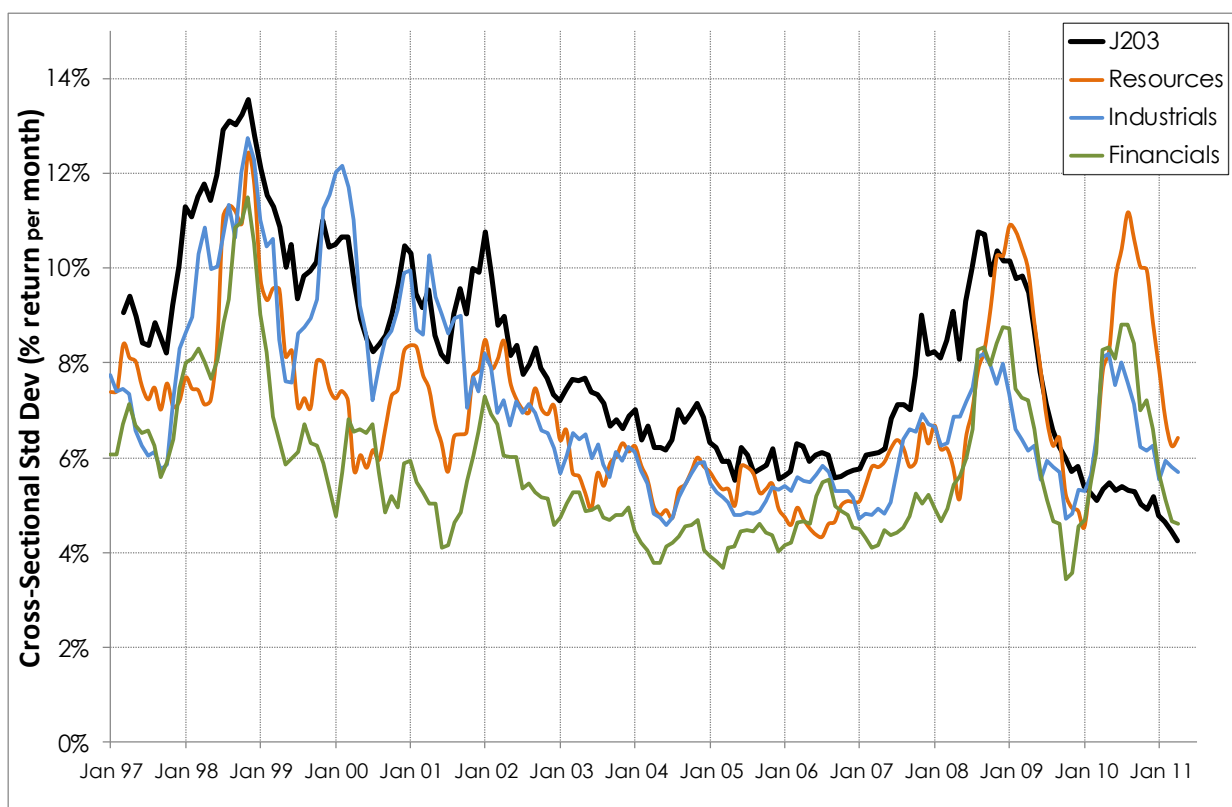
While the cross-sectional diversity of the index is relevant to general equity managers and investors, specialised sector portfolios will be concerned with the cross-sectional diversity of the sectors within the ALSI. It's not uncommon for investors to require their portfolio managers to be sector neutral and focus only on stock-selection within these sectors: the portfolio will have an active sector weight close to zero but active weights on a stock level

---

<sup>32</sup> Refer to Section 2.3 Equation 2.13 for the definition of effective number of stocks.

which differ from zero. For these management arrangements, the intra-sector dispersion is also of more concern than the dispersion of the market as a whole.

Figure 4.3 displays the weighted cross-sectional standard deviation over time of all three major sectors alongside that of the ALSI. Since all three sectors are subsets of the ALSI and by definition, less diverse, it's not surprising that their cross-sectional dispersion is generally lower than the ALSI itself and follows the same historic pattern.



**Figure 4.3: Cross-sectional standard deviation on the FTSE/JSE sectors over time**

Although the resource sector is by far the most concentrated of the three sectors (about 80% of its weighting in five stocks), it is the financial sector which has more frequently offered the lowest cross-sectional opportunities for the active manager historically. If, as Sapra (2008) argues, the size of the cross-sectional standard deviation is an indication of

the extent of active management that's possible in each of these sectors, it is important to note the relatively low opportunities offered to an active financial sector portfolio manager in South Africa as opposed to those of an industrial or resources portfolio manager. This difference in sectoral dispersion provides some support for the differences in the expected scale of active performance and active risk in these specialist sector portfolios across time.

#### **4.4 IMPLICATIONS OF CROSS-SECTIONAL VOLATILITY AND ITS CHANGES FOR PORTFOLIO MANAGEMENT AND MANDATE SETTING**

In professional portfolio management, the fund sponsor and/or consultant and the portfolio manager are both concerned with active return generation but can inadvertently contaminate their common objective by failing to consider how their collective decision-making corresponds with the prevailing market conditions.

The consultant or sponsor's role is to determine the fund objectives using asset/liability concerns, levels of funding, legislative properties and risk appetite amongst other criteria. In this way the fund sponsor sets out the investment objectives and restrictions so that all parties are clear on the road ahead. In fact, in relatively new legislation (PF130<sup>33</sup>), the requirements for the investment policy statement are specifically spelled out in "Principle 8: The Investment Performance of Fund Assets". A substantial part of this process is a negotiation of performance expectations which, in an active management context, involves setting target active risk levels for the fund. For the most part, the consultant/sponsor's

---

<sup>33</sup> Draft circular PF 130: Good governance of retirement funds, available online at <http://www.irf.org.za/Documents/pdf%20files/Legal%20Technical/0710%20-%20Draft%20Governance%20Circular%20PF130.pdf>.

decisions in this regard are independent of the portfolio manager's investment view and largely independent of current market conditions.

The portfolio manager's role is to have an investment view i.e. to forecast active returns on securities and assets. The point of interaction between these two parties is when the investment mandate, with its fund objectives and risk controls, meets the portfolio manager's portfolio construction process: the mechanism whereby the investment view is translated into an actual investment. By way of portfolio construction, the investment view, developed in the laboratory of news, information, models, history and experience, is translated in the best possible way into an allocation of money across a variety of available assets. This allocation must meet with a variety of constraints: net individual investments that are too short, too concentrated, too geared or traded too often while simultaneously aligning with particular styles and ensuring neutrality with others. This allocation must be achieved while maintaining a given level or band of active risk.

#### **4.4.1 Implications for mandate setting**

This section focuses particularly on the setting of active risk targets or expectations and its implications on active management in varying market conditions. Implementing an investment view with a particular active risk in mind requires a sufficient "carrying capacity"<sup>34</sup> for active risk which varies over time. "Carrying capacity" is a phrase borrowed from environmental science and indicates the number of individual organisms that can be sustained by an ecosystem without negatively impacting the organisms or their

---

<sup>34</sup> Yu (2007) refers to this concept as "alpha granularity".



environment. As such, this ecological notion is an apt analogy for the cross-sectional dispersion of an investment environment at any given time: without sufficient cross-sectional dispersion, high active weights cannot be achieved without leverage or substantial relaxation of the typical mandated portfolio constraints.

The starting point of this analysis is the so-called “unconstrained” portfolio, where the active manager is free to invest both long and short in a way that optimally reflects their investment view. Equation 2.7 (Section 2.1) describes the analytical solution to such an optimisation. Sapra (2008) makes use of a simplified, two-parameter covariance matrix in order to derive the relationship between the component parts of the portfolio solution: forecast return, active risk, active weights and the risk characteristics of the investment universe. The active risk of such a portfolio is proportional to three particular parameters: a) the active weights ( $\mathbf{w}_A$ ), b) the cross-sectional risk of the investment universe ( $\sigma_{cs}$ ) and c) the size of the investment universe ( $N$ ). Using this two-parameter simplification and rearranging the terms of Equation 2.7, Sapra (2008) derives the relationship described in Equation 4.2.

#### **Equation 4.2: Active risk components of an unconstrained portfolio**

$$\sigma_A \propto \mathbf{w}_a \sigma \sqrt{1 - \rho} \sqrt{N}$$

where

$\sigma_A$  is the target active risk of the portfolio;

$\mathbf{w}_A$  is a vector of the active weights or active investment positions making up the portfolio;

$N$  is the number of securities in the permitted investment environment;

$\sigma$  and  $\rho$  are the two parameters of the simplified covariance matrix (standard deviation and correlation respectively); and

$\sigma\sqrt{1-\rho}$  represents cross-sectional risk.

Cross sectional risk and the size of the investment universe, are a function of market conditions and the mandated investment universe. The previous section (Section 4.3) showed these parameters (particularly the former) to be time-varying and, unfortunately, outside of the control of either the fund sponsor or the portfolio manager once the portfolio domain has been decided. The active weights of a portfolio are the domain of the portfolio manager and represent their decisions while the target active risk is the expression of the fund sponsor's objectives in terms of the portfolio's aggression.

The relationship between the target active risk and the active weights are obvious. The more confident and aggressive the active manager in pursuing their investment view, the larger the active positions of the portfolio will be and consequently, the higher the active risk (all other things being equal). Much of the risk budgeting process and the active risk expectations are based on this simple principle. After all, the ultimate way in which a portfolio manager can express themselves is in their active positions. The fund sponsor would be wise to expect these to reflect both the view and the level of active risk on the part of the portfolio manager.

Equation 4.2 illustrates that the relationship between active risk and active weights must also involve the complex, multi-dimensional relationships between securities. In particular, in the simplified world of the unconstrained portfolio manager, the higher the

cross-sectional volatility in the investment environment, the more easily an active portfolio manager can acquire the required active risk. Conversely, when the cross-sectional variation “dries up” the active manager must implement larger active weights to maintain the active risk target.

Good betting strategy would increase the stakes when the opportunities are high and vice versa. Following this directive, the rational, optimal and unconstrained active portfolio manager would take large active positions when the market has high cross-sectional dispersion and small active positions when the cross-sectional variation is low. Consequently, the active risk of their portfolio would wax and wane with increases and decreases in the dispersion among stocks.

In practice, a widely varying active risk would be a concern for fund sponsors who, typically, anchor the active risk for their portfolios to a particular range or level, independent of market conditions. Fund sponsors may, for example, wish to maintain a certain active risk level in a portfolio in order to fit their particular long-term risk budgets or funding levels. Similarly, active managers can set their sights on a particular active risk quantum with which they define their particular lack of benchmark-fear<sup>35</sup>! According to Equation 4.2 then, the rational, optimal active portfolio manager, constrained to a rigid active risk, would behave contrary to good betting strategy when cross-sectional dispersion changes by increasing their active positions in their portfolios when

---

<sup>35</sup> “Benchmark-fear” is one of the hypotheses that Bernstein (1991) put forward as a possible explanation for the decline in fund performance dispersion before the 1990’s.

opportunities are low and decreasing their bets when the market shows greater dispersion in security returns.

As shown in Figure 4.1, this capacity of the ALSI to deliver active risk-taking opportunities has been reducing steadily since the 1998 recovery to 2007. Given the relationship between cross-sectional volatility and active risk, portfolio managers and sponsors should have become accustomed to steadily decreasing active risk in their portfolios over that period. But if sponsors and managers required constant levels of active risk throughout this period, active managers would have become accustomed to increasing active weights and, for managers whose mandates allow the incurring of liabilities, increased gearing. According to Tobin's Separation Theorem (Tobin (1958)) and its application in an active management framework, if you can't achieve the same amount of active risk as you've learnt to expect historically in a low opportunity market, the most efficient solution is to gear up.

Khandani and Lo (2007) put forward the un-testable hypothesis that this kind of behaviour by hedge fund managers likely precipitated the hedge fund crisis of August 2007: cross-sectional volatility had been diminishing for years (as were the returns to over-crowded strategies) but clients and managers resisted allowing their active risk and return expectations to wane in response. Consequently, the strategies remained the same but the gearing increased as the carrying capacity decreased, leaving many hedge funds in an irresponsibly vulnerable position when the credit crunch came.

Gearing and credit-risk are less of a concern in the realm of long-only portfolio management (typical of South African pension fund mandates). However, long-only managers are not immunised from the effects of decreasing cross-sectional volatility. The long-only active manager, like his/her unconstrained counterparts, can only maintain a constant active risk in a market with decreasing dispersion by increasing their active weights. However, the long only manager will, at some point, for some assets, meet the boundary of the negative active weights possible for securities that can only be held long. When all the opportunities to express their investment view without corruption have been used up, the long-only manager must increase active weights in areas where they *can* rather than where their investment view implies they *should*. Large fund size, a small and concentrated investment universe, and several additional mandated risk controls and limits into the mix and the distortion of the investment view is exacerbated.

In this way, when the decision makers set portfolio mandates and objectives with rigid, under-researched active risk targets and portfolio constraints, the translation of an investment view into a portfolio of investments becomes distorted with changes in the “carrying capacity” of their investment universe. In particular, when the cross-sectional variation in the market reduces, the resulting portfolio can be too active for the carrying capacity of its investment universe and, worse still, inconsistent with the manager’s investment view. Furthermore, when the portfolio manager’s performance is assessed, the ex post performance of the portfolio might bear little resemblance to the portfolio managers’ intended source of active return. (The following section deals with some of the implications of the time varying nature of cross-sectional variation with regards to performance monitoring.)

Therefore, when setting the investment objectives of a fund, fund sponsors should be mindful of the tools at each decision maker's disposal as well as the conditions of the market at each point in time. Fund sponsors can set target active risk levels and portfolio managers can widen the active weights but, if these requirements and the portfolio's constraints aren't matched to the market's capacity to deliver active return (i.e. the cross-sectional dispersion on the market), the investment process can be considerably contaminated.

#### **4.4.2 Mandate compliance and forecast errors**

When fund sponsors monitor mandate compliance and whether their selected managers are delivering according to the sponsors' expectations, they will typically receive performance reports using forecast active risk and information ratios as an indication of the positioning of the portfolio. Using an established and accepted risk model representing the investment universe (usually provided by an independent service provider and modelled for the most part on ex post stock behaviour), the ex ante active risk will be calculated by multiplying the current active positions in the portfolio (which are required to be mandate compliant) by the covariance matrix of forecast active returns (refer Equation 4.3<sup>36</sup>).

---

<sup>36</sup> As mentioned in Section 2.1, this is essentially just the ex ante version of Equation 2.4

**Equation 4.3: Ex ante active risk estimation**

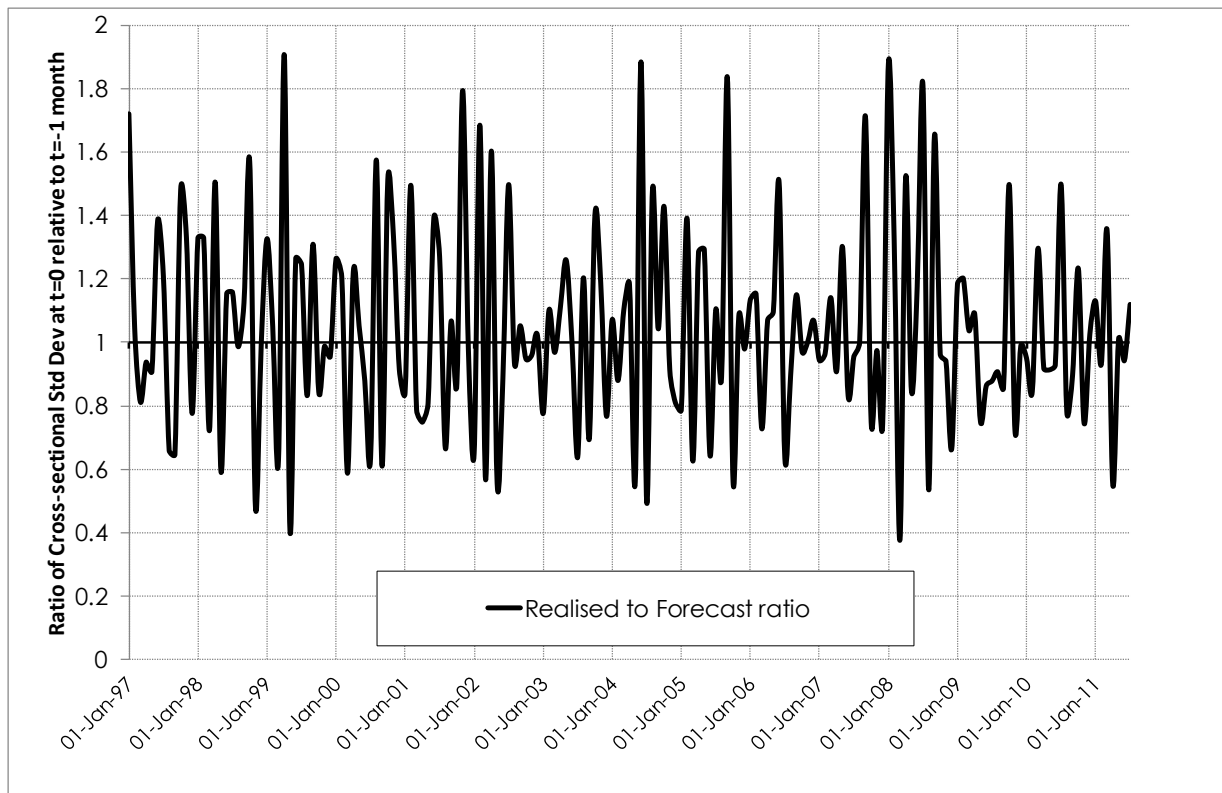
$$\widehat{\sigma}_a = (\mathbf{w}_A' \boldsymbol{\Sigma} \mathbf{w}_A)^{\frac{1}{2}}$$

where  $\widehat{\sigma}_a$  is the ex ante active risk forecast; and

$\boldsymbol{\Sigma}$  is the covariance matrix of forecast returns

Many fund sponsors and active managers in South Africa were taken by surprise by a growing increase in their active risks over 2007 and continuing into 2008. In several cases, mandated risk levels were enforced and active portfolio positions were cut in order to bring portfolios back to compliant levels. The source of these dramatic increases in ex ante active risk forecasts lay in the steep increase in the ALSI's cross-sectional risk from 2007 to 2008 evident in Figure 4.1. In the simplified expression of the relationship between realised active risk and forecast active risk, developed in Sapra (2008) (refer Equation 2.17), the forecast error in active risk is shown to be directly proportional to the forecast error in cross-sectional standard deviation. By implication then, if portfolio managers and sponsors are neglecting to monitor the changes in cross-sectional standard deviation by using long-term longitudinal risk models, the anticipated ex ante active risk can be surprisingly different to the realised active risk and insensitive to changes in the active management environment.

To illustrate the extent of this potential “shock” to active risk estimations, Figure 4.4 shows the ratio of the realised cross-sectional standard deviation to its one month lag. This provides an indication of the error multiple of forecast active risk from month to month (for fund sponsors who receive monthly performance reports).



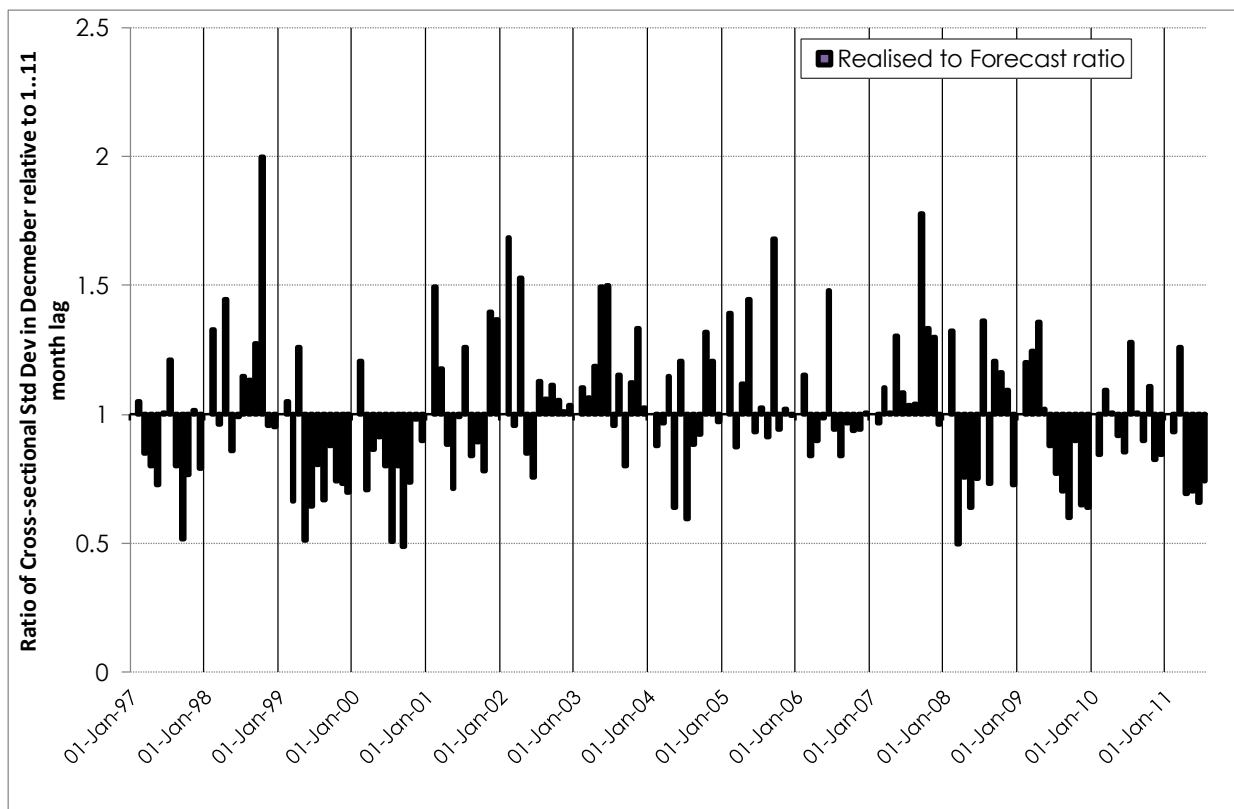
**Figure 4.4: Ratio of realised cross-sectional standard deviation to its one month lag**

The extent to which the realised active risk differed from the targeted active risk, implied by this relationship between realised and forecast cross-sectional standard deviation, varied between 40% to nearly double the active risk realised compared to the intended active risk of the portfolio. By implication, when using active risk as a measure of mandate compliance, the potential decision making consequences are severe. If the realised active risk is considered unexpectedly high, the portfolio manager could be required to reduce active positions which, under different market circumstances, would have been quite acceptable and vice versa.

Figure 4.5 illustrates the potential drift in the accuracy of the forecast active risk over a calendar year using increasing lags from each January in the ratio of realised cross-



sectional standard deviation. This illustration attempts to show, on a calendar year basis, the effect of changes in cross-sectional risk on the active risk statistic.



**Figure 4.5: Realised cross-sectional standard deviation relative to increasing lags**

Figure 4.5 demonstrates that the relative size of the shock to active risk can be anything from half to double the original estimate as a result of changes in cross-sectional volatility within a calendar year. There were times within 2008 where the realised active risk would have been half the size implied by the January cross-sectional risk of the market, making asset managers appear to have radically decreased their risk appetites when in fact the equity market was simply changing the spread in its investment offerings.

The implication of this relationship between cross-sectional market conditions and active risk forecasts is that sponsors should expect the active risk estimate of any portfolio to

vary under varying market conditions. A change in an ex ante active risk could be an indication of changing active weights or portfolio aggression, which is why this monitoring is important. However, it is also very possible that changes in the ex ante active risk of a portfolio are simply a reaction to changes in market conditions. Sponsors and their managers should therefore exercise caution when reacting to short term changes in this estimate on their performance reports by closing out active positions.

## **4.5 CONCLUSIONS**

Cross-sectional volatility and its changes are a good measure of the investment opportunity set for active managers. The more dispersed the returns of investments are, the more opportunities there are for active managers to perform differently (preferably better!) from their competitors and the benchmark.

Cross-sectional volatility has changed substantially over time as have the opportunities for superior active performance and there is no reason to expect that the cross-sectional risk of the market won't continue to vary over time. The relationship between this changing cross-sectional volatility and the dispersion of active portfolio managers further illustrates the importance of cross-sectional volatility to active management.

Ex ante, cross-sectional volatility must be considered hand-in-hand with risk limits and active risk targets when mandates are set or monitored. The higher the cross-sectional volatility, the greater the opportunity for active risk taking, all other things being equal. Conversely, to remain efficient, active risk taking should be reduced during periods of low cross-sectional dispersion.

Furthermore, when fund sponsors monitor changes in an ex ante active risk they should bear in mind that changes in the active risk forecast of a portfolio could be a reaction to changes in market conditions and not the result of changes in the active positions of the fund. Sponsors and their managers should therefore exercise caution when reacting to changes in active risk estimates, mindful of the relationship between cross-sectional dispersion and active risk.

## **Chapter 5 : Varying cross-sectional volatility in asset returns and the implications for ex post performance measurement**

### **5.1 SUMMARY**

Ex post investment fund performances are assessed over multiple periods in order to deduce skill on the part of their managers. The extent of the dispersion among competing funds' performances varies substantially over time, resulting in heteroscedastic time series of fund performance. This heteroscedasticity impedes the use of standard parametric methods (e.g. t-statistics, Sharpe or Information Ratios) for detecting significant superior performance and renders ordinary least squares (OLS) regression inappropriate. This chapter recommends correcting the ex post performance of competing portfolio managers for the varying cross-sectional risk of their investment environment in order to more accurately and fairly assess the extent of their success and skill.

### **5.2 INTRODUCTION**

This chapter focuses on fund-level performance measurement and omits analysis which requires knowledge of the composition of the underlying portfolios. The chapter continues with an empirical demonstration of the relationship between the dispersion of equity performance and the dispersion of equity fund performance using South African General Equity mutual funds as a sample case. Next, a test for heteroscedasticity as a function of the dispersion of benchmark assets is applied to the time series of fund returns. Following this test for heteroscedasticity, a weighted-regression is implemented and the effect of such an adjustment on the heteroscedasticity is measured. The analysis demonstrates the difference in performance ranking as a result of such a correction. In

the final section of this chapter, a similar weighting technique is applied to simple performance measures such as the annual active fund returns and the risk-adjusted ratio of fund returns to illustrate the differences in performance rankings that result. The conclusions to this chapter follow.

### **5.3 ANALYSIS**

In this section, the relationship between the dispersion of equity portfolio performance and the cross-sectional dispersion in the equity market is shown empirically and corrections for this heteroscedasticity are applied.

#### **5.3.1 Data and sampling**

Our sample comprises all the South African Domestic Equity mutual funds<sup>37</sup> that might reasonably use the ALSI as their investment universe and benchmark. This sample includes most of the General Equity classification of South African registered unit trusts (mutual funds) but excludes Islamic Sharia funds, index funds and funds of funds<sup>38</sup>. Index funds are excluded from this analysis because their objective is to avoid incurring either active risk or return and, as such, their performance objectives differ to those of their peers

---

<sup>37</sup> Segregated funds with the same management and mandate (and therefore almost identical fund performance histories) were represented by the primary (oldest) listing to avoid double counting.

<sup>38</sup> The performance of funds of funds would be sensitive to changes in the cross-sectional volatility of the market but, because they are assumed to allow for an added element of diversification, their sensitivity is expected to be on a different scale. Funds of funds performance should arguably be corrected with the cross-sectional volatility of their own market, namely the cross-sectional volatility of the funds in which they are able to invest which, due to diversification of risk, would exhibit lower cross-sectional dispersion than equity returns.

in this group. Specialist funds are excluded from this analysis although they are also expected to exhibit a relationship between their performance dispersion and the dispersion of the market. Specialist funds such as sector-specific or Islamic funds exclude certain stocks from their investment universe, thereby exhibiting materially different performance history to the rest of the equity funds which in turn would exacerbate the dispersion of fund returns used in this section for illustration. The dispersion among specialist funds should be compared instead to their own investment universe to illustrate the same point. The choice of the General Equity category for this analysis enables the illustration of dispersion across a large sample of peer funds. The necessity for similar adjustments to performance metrics is no less valid in smaller and more specialised fund types.

The fund performances used here comprise the monthly return<sup>39</sup> history (excluding fees) of all the sample of funds from December 2000 to January 2011<sup>40</sup> comprising a sample of 24 funds in 2001 growing to 61 funds in 2011. The total return of the ALSI and its constituent stocks over the same period were also required for the analysis that follows, as well as their weighting<sup>41</sup> in the index at each point in time.

### **5.3.2 Fund performance dispersion vs. benchmark dispersion**

Figure 5.1 charts the dispersion of the General Equity funds' returns each month (max, min, upper and lower quartile) on the same axis as the ALSI's return. The bar chart below

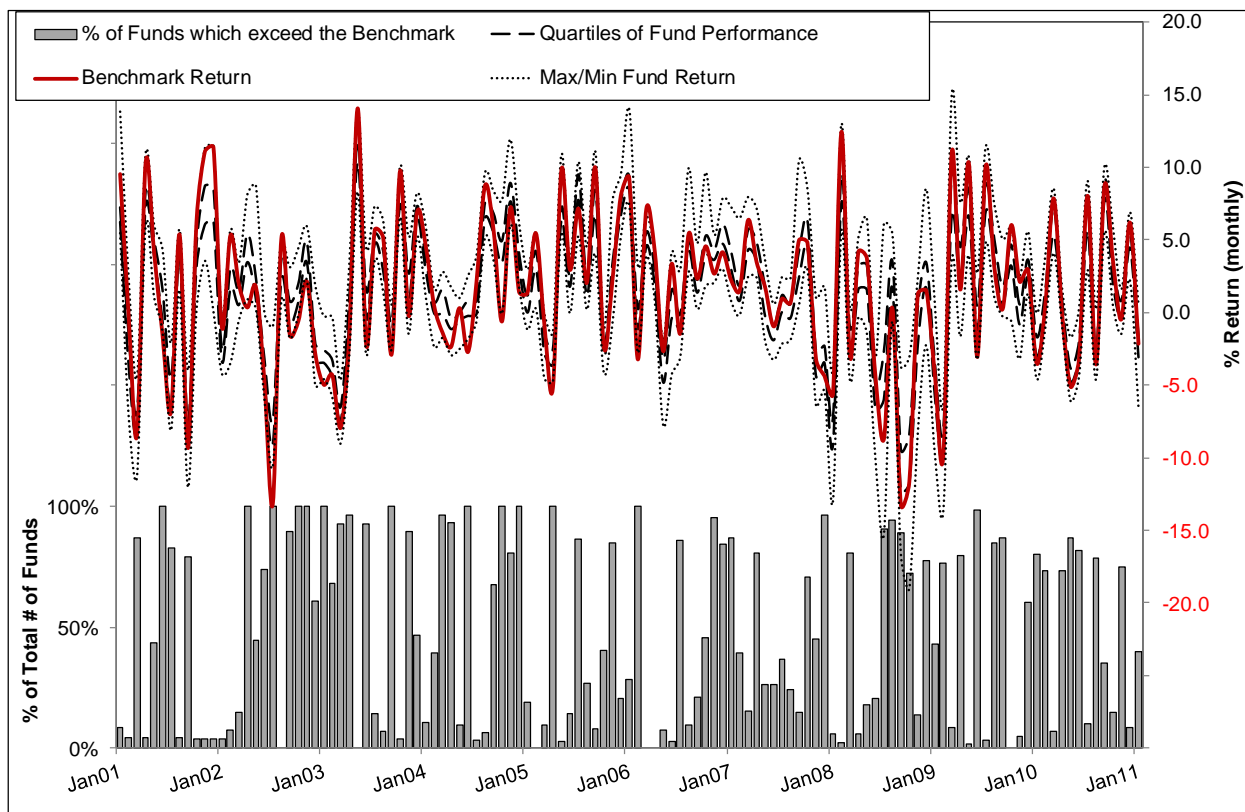
---

<sup>39</sup> Changes in net asset value (NAV) of the fund

<sup>40</sup> Source: Moneymate

<sup>41</sup> The ALSI is a capitalisation weighted index. There is no requirement that the benchmark used in such a study be capitalisation-weighted, only that it represents the investment universe of the funds to which it is applied in a passively managed way.

these returns represent the proportion of all available General Equity funds which achieved greater return than the ALSI in each month.



**Figure 5.1: Fund performance dispersion around the benchmark**

Figure 5.1 illustrates the obvious relationship between fund performance and the time-changing performance of the benchmark: the spread of fund performances surrounds the benchmark's performance and varies in a similar way. In this way, Figure 5.1 emphasises the nature of active fund management: fund returns are gleaned directly from the universe of investable assets by investing in varying proportions in the same set of assets. No value is created by the fund manager, who can only earn or avoid profit or loss on the part of the available assets in varying proportions and with varying success. Although the funds derive their performance from benchmark assets, as discussed in Chapter 2, Figure 5.1 shows that the benchmark performance is seldom central to the distribution of realised fund performances. Over the time period included in this study, an average of 47% of the

funds delivered performance in excess of the benchmark every month (40% of the funds did the same on an annual basis). Although the average in this sample is close to 50%, the variation in the proportion of funds which successfully “beat the benchmark” every month is substantial, as illustrated by the bar chart in Figure 5.1 emphasising the non-centrality of benchmark performance.

Figure 5.2 illustrates the less well-known relationship between the dispersion of fund performances (specifically active or excess-of-benchmark performance) and the dispersion of benchmark assets at each point in time. In Figure 5.2, all the time series depicted represent dispersion or spread. The cross-sectional dispersion in fund performances is represented in Figure 5.2 by three different metrics: range (plotted against the secondary axis on the right hand side), inter-quartile range and standard deviation. The dispersion of benchmark assets is plotted in red against the secondary axis on the right and is represented by the weighted cross sectional standard deviation<sup>42</sup> of total returns on the securities underlying the benchmark, in this case the ALSI.

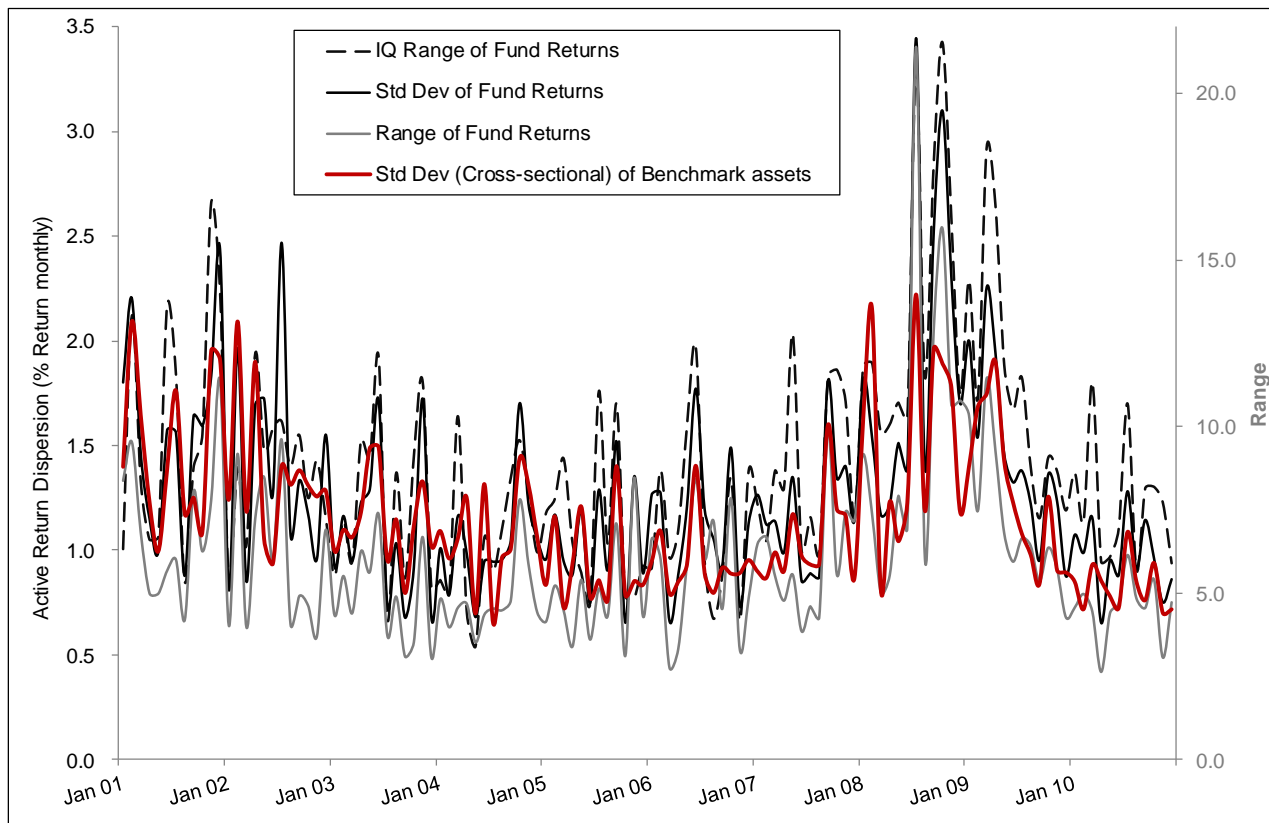
Figure 5.2 illustrates two important features in fund performance. Firstly, the changing nature of cross-sectional standard deviations of these portfolio performances each month can be seen in Figure 5.2, peaking in 2002 and again in 2008 and 2009. By implication, this figure demonstrates the waxing and waning of the competitive field for active managers even after correcting their funds’ performances for the benchmark’s varying performance. Figure 5.2 is thus a visual confirmation of the presence of heteroscedasticity in peer-group fund performances over time. Secondly, Figure 5.2

---

<sup>42</sup> As per Equation 4.1



shows the relationship between the varying dispersion in benchmark assets over time and the varying dispersion of competing fund performances over time. This relationship implies an easily and objectively measurable source of the heteroscedasticity in fund performance thereby enabling a simple adjustment to this heteroscedasticity, independent of the sample of funds.



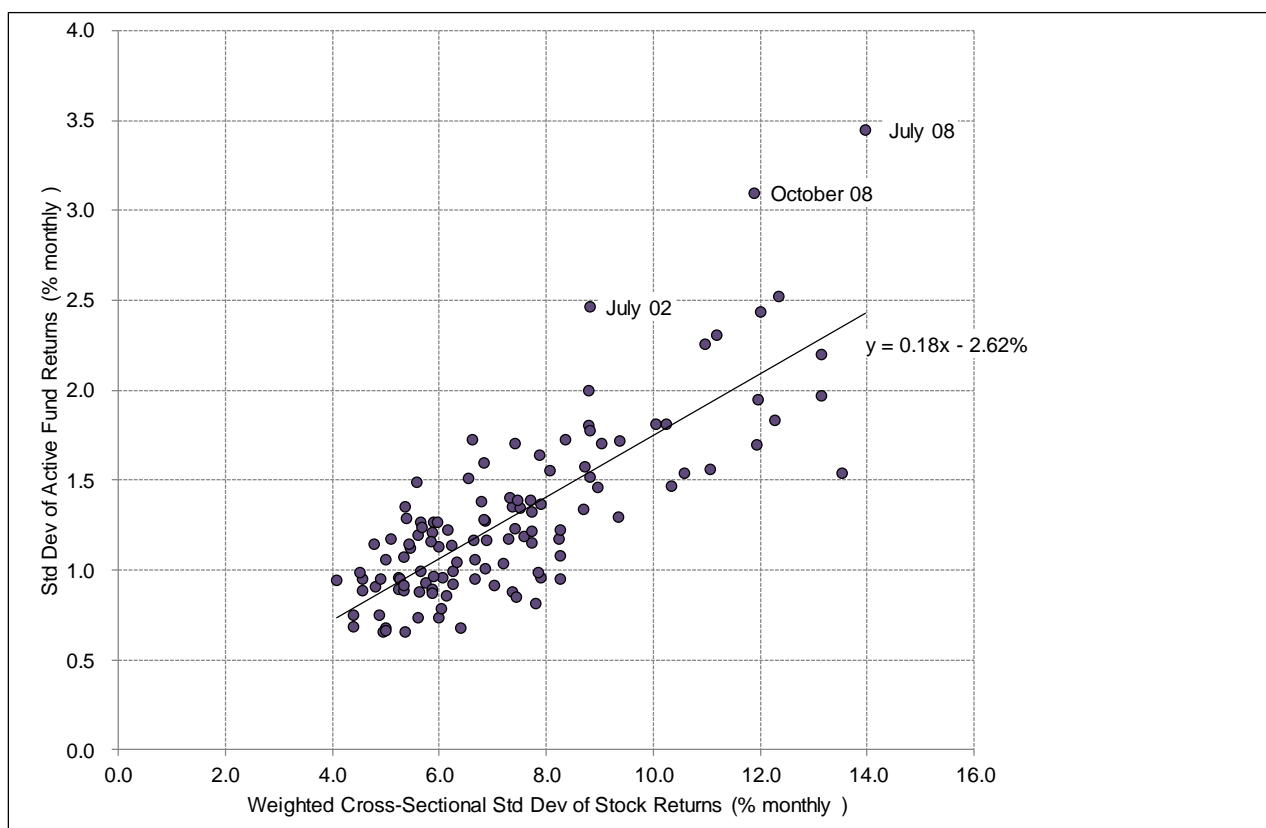
**Figure 5.2: Cross-sectional dispersion on the ALSI and dispersion in the active returns of General Equity mutual funds**

This relationship is shown more clearly in Figure 5.3, a scatter plot of the dispersion in fund performance against benchmark asset dispersion. The coefficient of determination between the monthly cross-sectional standard deviation of portfolio active returns<sup>43</sup> and

---

<sup>43</sup> The biases as pointed out in Sharpe (1991) are noted in terms of using averages and standard deviations when we are unable to weight the funds according to their size. Parametric methods will likely create a small-cap bias in the measure of fund dispersion.

that of the ALSI is approximately 65% over this 10 year period. In the US, using a larger sample of mutual funds from 1981 to 2000, De Silva *et al.* (2001) found that 91% of the changes in the annual dispersion in of all active equity portfolios could be explained by changes in the dispersion in the equity market. The authors expected portfolio dispersion to be of a lower magnitude than security dispersion given the diversification of fund holdings and empirically they found a fairly stable ratio of security dispersion to portfolio dispersion of about 4:1. The analysis of monthly portfolio data in this chapter finds a less stable relationship between portfolio dispersion (as measured by the cross-sectional standard deviation) and the ALSI's dispersion (R-squared of 65%) with a median ratio of about 6:1 which has varied substantially over this 10 year period.



**Figure 5.3: Scatter plot of the cross-sectional standard deviation of fund performance and the weighted cross-sectional standard deviation of the benchmark assets**

Empirically, the larger and more homogenous the sample of competing funds, the clearer the relationship between fund dispersion and benchmark dispersion is likely to appear. Analytically, it's clear from the evidence presented here that dispersion in fund returns will vary with the weighted dispersion of underlying assets (refer Yu and Sharaiha (2007) and Gorman *et al.* (2010)). It follows that multi-period competitive performance should be viewed with this external source of heteroscedasticity in mind in order to more accurately and fairly assess relative success or failure of the fund manager's actions.

### 5.3.3 The influence of benchmark dispersion on OLS regression metrics

OLS regression assumes homoscedasticity and regression-based fund performance metrics will accordingly be misstated as a consequence of the heteroscedasticity in the time series of fund returns. In professional asset management parlance, the intercept of a fund's regression against its benchmark, "alpha" is considered to be an ex post estimate of manager skill while the slope of the same regression indicates leverage (refer Equation 5.1).

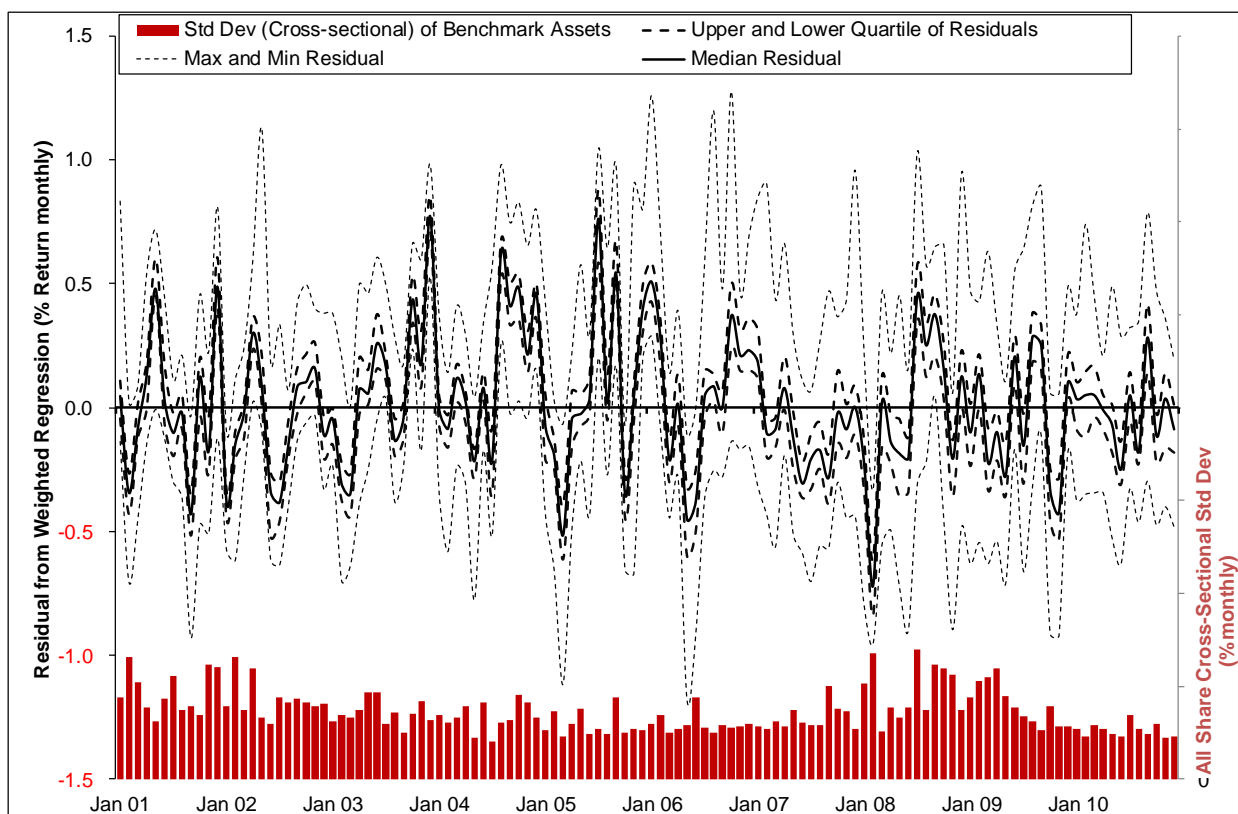
#### Equation 5.1: OLS performance analysis

$$R_{p,t} = \hat{\alpha}_p + \hat{\beta}_p R_{b,t} + \varepsilon_{p,t}$$

where  $\hat{\alpha}_p$  and  $\hat{\beta}_p$  are fitted regression coefficients for the intercept and slope respectively; and

$\varepsilon_{p,t}$  are the residuals which are assumed to be distributed random normal with a constant variance ( $\sigma_\varepsilon^2$ ) and zero mean.

To illustrate, Equation 5.1 was applied to each of the funds in the sample using the ALSI as the common benchmark portfolio and using all the available history of each fund's performance. Figure 5.4 is a plot of the five number summary of the cross-section of the resulting residuals ( $\varepsilon_{p,t}$ ) of each fund's regression over time with the dispersion of the benchmark assets plotted as a bar chart below as a reference. Figure 5.4 illustrates the effect that high benchmark dispersion had across all 60 of these regressions: the spread in the residuals were generally higher across funds when benchmark dispersion was high indicating less estimation accuracy for these periods.



**Figure 5.4: Plot of the distribution of OLS residuals across all General Equity mutual funds**

### 5.3.3.1 Test for heteroscedasticity

To test for the presence of heteroscedasticity in these OLS regressions described by Equation 5.1, we test each of the 61 regressions (the regression applied to each fund).

For these purposes, we use the Breusch-Pagan test. In particular, we test the hypothesis that the variance of the residuals in Equation 5.1 for each fund are not constant but are in fact an unknown function of the time series of cross-sectional standard deviation of the benchmark assets i.e.  $\varepsilon_{p,t} \sim N(0, \varphi \sigma_{cs,t}^2)$  where  $\varphi$  is an unknown constant and  $\sigma_{cs,t}^2$  is the cross-sectional variance of the benchmark assets at each point in time,  $t$ .

The Breusch-Pagan test in this case is a test of the significance of the relationship of the squared residuals from the application of with the realised cross-sectional variance of the benchmark ( $s_{cs,t}^2$ ). This is performed by way of an F-test for a second regression of the squared residuals against the cross-sectional variance (refer to Equation 5.2).

**Equation 5.2: Breusch-Pagan regression to test for heteroscedasticity related to benchmark dispersion**

$$\frac{\varepsilon_{p,t}^2}{\varepsilon_p^2} = \hat{a} + \hat{b}s_{cs,t}^2 + \gamma_t$$

where  $\overline{\varepsilon_p^2}$  is the average squared residual;

$\hat{a}$  and  $\hat{b}$  are fitted regression coefficients; and

$\gamma$  is the residual term of this regression.

Of the 60 time series of squared residuals to which this regression in Equation 5.2 was applied, 86% were significant with p-values less than 5%. That is, 51 of the 60 funds' time series of OLS residuals showed a significant linear relationship with the dispersion of benchmark assets. These results provide strong empirical support for the need to apply a weighted-regression to correct for heteroscedasticity in fund performance, using the

dispersion of benchmark assets as the correction factor. The first five columns of Table 5.3 summarise these results.

## **5.4 CORRECTING PERFORMANCE MEASUREMENT FOR HETEROSCEDASTICITY**

### **5.4.1 Weighted regression**

Weighted OLS regression (White, 1980) is a well-used solution to the problem of heteroscedasticity in residuals. Most often, the source of the heteroscedasticity is unknown and the weights applied to the time series used in the weighted regression must be estimated from the independent variables themselves. In this context, the likely source of the heteroscedasticity for which we want to correct is the cross-sectional dispersion of benchmark assets which can be objectively and independently measured simultaneously with the measurement of fund performance and is not subject to sampling concerns. Furthermore, in a management context where the performance of fund managers is assessed and compared using regression metrics, the use of an external variable which is not subject to the influence of the same managers makes for a reliable and objective correction for heteroscedasticity in the OLS regression.

Essentially, the weighted regression methodology gives greater importance to more reliable data (data with lower variance) and less importance to data sampled from distributions with greater variance. In this context, fund performances which were earned from less disperse markets will carry a heavier weight and vice versa, allowing true skill on the part of the manager to be more detectable amidst changing market conditions. In other words, a 1% fund performance is more indicative of a skilful manager when it is

earned from a market which exhibited low dispersion in asset returns than it is when earned from a market with greater dispersion and wider ranging return opportunities.

The weighted OLS regression that is envisaged here is described in Equation 5.3. Each term (including the residual) is divided by the cross-sectional standard deviation of the market.

### Equation 5.3: Weighted OLS regression

$$\frac{R_{p,t}}{s_{cs,t}} = \hat{\alpha}_p^* \frac{1}{s_{cs,t}} + \hat{\beta}_p^* \frac{R_{b,t}}{s_{cs,t}} + \frac{\varepsilon_{p,t}}{s_{cs,t}}$$

As a result, the heteroscedastic residual described from the OLS regression (Equation 5.1) will be transformed by the weighted regression in Equation 5.3 to be

homoscedastic i.e. the residual from Equation 5.3 ( $\frac{\varepsilon_{p,t}}{s_{cs,t}}$ ) will have a constant variance (

$$\frac{\varepsilon_{p,t}}{s_{cs,t}} \sim N(0, \phi).$$

Also note that, instead of fitting an intercept term as in Equation 5.1, the weighted regression in Equation 5.3 uses the inverse of the cross-sectional standard deviation as a second explanatory variable. In a fund performance context, the interpretation of this fitted coefficient would be comparable to the stock-picking alpha of the OLS described by Equation 5.1.

When this weighted regression is applied to the same sample of 60 funds with performance time series of various lengths, the overall size of the residuals are obviously reduced. More importantly, only 10 of the 60 funds produce significant Breusch-Pagan test results after the weighting adjustment demonstrating that the heteroscedasticity in fund performances has been effectively corrected using this weighting procedure. Table 5.3 summarises the regression results before and after the weighted adjustment is applied.

#### ***5.4.1.1 Performance rankings using regression metrics***

When fund performances are compared across competitors, the intercept term of the regression (commonly referred to as “alpha”) is attributed to stock-picking success/failure on the part of the manager. The previous section showed that the OLS regression (Equation 5.1) is flawed on account of the heteroscedasticity in the time-series of fund and benchmark returns and that a weighted regression can be used to address this problem. In this section we illustrate the impact of the weighted-regression methodology (Equation 5.3) on the relative rankings of estimated stock-picking ability (i.e. the intercept term).

Table 5.1 comprises the intercept from both the weighted and un-weighted regressions over the five-year period ending January 2011, using monthly fund and benchmark total returns for the full period. Funds with a shorter performance history were excluded from this analysis. These two estimates of stock-picking skill are also ranked and their percentile given so that they can be competitively evaluated, as they would by prospective clients/investors. The funds are listed in order of descending stock-picking skill as measured by the weighted regression.



**Table 5.1: Ranked “alphas” using un-weighted and weighted regressions  
(5 year monthly return history to January 2011)**

Name	OLS			Weighted OLS		
	Alpha	Rank	Percentile	Alpha	Rank	Percentile
Sasfin Twenty Ten	-0.04	20	52%	0.83	1	100%
Kagiso Equity Alpha	0.37	3	91%	0.69	2	79%
ABSA Select Equity	0.46	1	100%	0.65	3	71%
Foord Equity R	0.19	11	74%	0.64	4	70%
Hermes Equity A	0.20	8	75%	0.63	5	69%
Allan Gray Equity A	0.26	4	81%	0.61	6	67%
Nedgroup Inv Rainmaker A	0.23	5	78%	0.61	7	66%
Prudential Equity A	0.37	2	92%	0.60	8	65%
SIM General Equity R	0.19	9	75%	0.58	9	62%
Oasis General Equity	0.21	6	76%	0.56	10	58%
Old Mutual Growth R	0.13	13	68%	0.55	11	57%
Nedgroup Inv Quants Core Eq A	0.11	14	67%	0.47	12	45%
ABSA General R	0.14	12	70%	0.47	13	45%
Element Earth Equity A	0.19	10	74%	0.45	14	41%
FNB Growth	-0.09	27	47%	0.43	15	39%
Coronation Equity R	0.20	7	75%	0.43	16	39%
Analytics Managed Equity A	0.02	15	58%	0.43	17	38%
Old Mutual Top Companies R	0.00	16	56%	0.40	18	33%
Old Mutual Active Quant Equity A	-0.01	17	55%	0.40	19	33%
Valugro General Equity	-0.08	25	48%	0.39	20	32%
Cannon Equity A	-0.09	26	47%	0.39	21	32%
STANLIB MM Equity A1	-0.06	24	50%	0.33	22	22%
Inv Solutions MM Equity A	-0.04	21	52%	0.31	23	20%
Coris Capital General Equity	-0.03	18	53%	0.31	24	20%
MI-PLAN IP Beta Equity B2	-0.05	23	51%	0.31	25	19%
Old Mutual Investors R	-0.04	22	52%	0.30	26	19%
PSG Equity A	-0.14	30	42%	0.29	27	17%
STANLIB Equity R	-0.41	39	16%	0.29	28	16%
Investec Active Quants A	-0.16	33	41%	0.28	29	16%
Community Growth Equity	-0.03	19	53%	0.27	30	14%
RMB Equity R	-0.19	35	37%	0.27	31	13%
Nedgroup Inv Equity R	-0.15	31	42%	0.24	32	9%
Gryphon All Share Tracker	-0.11	29	45%	0.24	33	8%
Indequity Technical	-0.11	28	46%	0.24	34	8%
Maestro Equity A	-0.23	37	34%	0.20	35	3%
Investec Equity R	-0.16	32	41%	0.20	36	2%
Sasfin Equity	-0.18	34	39%	0.19	37	2%
Interneuron Capital Equity	-0.37	38	21%	0.19	38	1%
Metropolitan General Equity	-0.20	36	37%	0.18	39	0%
STANLIB SA Equity R	-0.58	40	0%	0.18	40	0%

Since all the funds are affected by the variation in the market's dispersion, it's not surprising that the weighted and unweighted rankings presented in Table 5.1 are not generally very different. However, there are some notable exceptions which are highlighted in the table. Sasfin's Twenty Ten fund, for example, has one of the highest unweighted performance volatilities of the 40 funds with a five year track record. One of the reasons for this is that Sasfin Twenty Ten delivered some of the worst performance amongst its peers during the times when the benchmark had the highest cross-sectional dispersion and vice versa. The OLS estimate of alpha ranked this fund poorly (20<sup>th</sup> amongst its 40 peers) because such a regression attributes the same value to all these monthly performance events as any other month in the market. The weighted regression, however, under weights Sasfin's performance during these high dispersion events and overweights them during the low dispersion events and results, in the case of Sasfin's Twenty Ten fund, in an estimate of alpha which is far more competitive (ranked first amongst its peers). For this fund, the failure to correct for heteroscedasticity could have had a material effect on its capacity to attract new business and retain existing clients.

Other examples of funds that, under the adjusted regression, changed more than 10 places in their competitive ranking were FNB Growth, Stanlib Equity and Community Growth Equity.

#### **5.4.2 Weighted Sharpe Ratios**

Regression is a fairly sophisticated measurement technique for assessing fund performance after the fact and the business media more typically publishes average returns and/or the ratio of return to longitudinal risk. However, as discussed in Chapter 2,

these statistics have their origin in a t-statistic and are effective in a homoscedastic population. We therefore recommend (as did De Silva *et al.* (2001)) that longitudinal statistics (averages, standard deviations etc) be applied to a weighted time series of performance where the weightings are the inverse of the benchmark's cross-sectional standard deviation at each point in time. This technique allows for the underweighting of the more uncertain, high market dispersion events and the overweighting of the fund performances derived from less disperse markets, correcting for heteroscedasticity and improving the power of the statistic to detect superior skill on the part of the fund manager.

Table 5.2 lists the ratios of average monthly realised return to standard deviation of the same per fund over a five-year performance period. This ratio is applied to both a weighted (correcting for heteroscedasticity) and unweighted time series and the funds are listed in descending order of this ratio as applied to the weighted time series. Once again, both of these ratios are ranked and their percentile given in order to compare the relative success of these managers in their peer group.

As with Table 5.1, the rankings of the success of these fund managers using this ex-post form of the Sharpe Ratio does not differ markedly overall whether a weighted or un-weighted ratio is used. However, there are a number of exceptions: four funds in particular have been highlighted in Table 5.2 whose rankings have been altered by ten or more places by applying the weighting procedure to their time-series of returns.

**Table 5.2: Ranked Sharpe Ratios using un-weighted and weighted time series (5 year monthly return history to January 2011)**

Name	Unweighted			Weighted		
	Ratio	Rank	Percentile	Ratio	Rank	Percentile
ABSA Select Equity	0.23	1	100%	0.38	1	100%
Kagiso Equity Alpha	0.23	2	99%	0.37	2	94%
Nedgroup Inv Rainmaker A	0.19	11	66%	0.36	3	87%
Allan Gray Equity A	0.19	9	68%	0.35	4	79%
Prudential Equity A	0.22	3	90%	0.35	5	78%
Hermes Equity A	0.19	10	68%	0.34	6	74%
Oasis General Equity	0.20	7	71%	0.34	7	71%
Element Earth Equity A	0.16	25	45%	0.33	8	70%
ABSA General R	0.18	14	61%	0.33	9	70%
SIM General Equity R	0.21	5	82%	0.33	10	68%
Foord Equity R	0.19	8	70%	0.33	11	66%
Old Mutual Growth R	0.20	6	72%	0.32	12	61%
Nedgroup Inv Quants Core Eq A	0.19	13	62%	0.32	13	60%
Coronation Equity R	0.21	4	84%	0.30	14	50%
Analytics Managed Equity A	0.17	18	53%	0.30	15	48%
Old Mutual Top Companies R	0.18	16	56%	0.29	16	45%
Old Mutual Active Quant Equity A	0.17	21	51%	0.29	17	45%
FNB Growth	0.16	26	42%	0.29	18	44%
Valugro General Equity	0.17	20	52%	0.29	19	41%
Community Growth Equity	0.17	17	54%	0.29	20	41%
MI-PLAN IP Beta Equity B2	0.17	23	50%	0.28	21	37%
Sasfin Twenty Ten	0.16	27	41%	0.28	22	35%
Inv Solutions MM Equity A	0.15	32	34%	0.28	23	35%
STANLIB MM Equity A1	0.14	35	29%	0.28	24	35%
Investec Active Quants A	0.18	14	61%	0.28	25	35%
Old Mutual Investors R	0.17	19	53%	0.28	26	35%
Gryphon All Share Tracker	0.19	12	64%	0.28	27	34%
Cannon Equity A	0.17	22	50%	0.28	28	34%
Coris Capital General Equity	0.17	24	48%	0.27	29	32%
Indequity Technical	0.15	29	38%	0.26	30	25%
RMB Equity R	0.14	33	31%	0.26	31	22%
Investec Equity R	0.14	34	30%	0.25	32	19%
PSG Equity A	0.15	28	38%	0.25	33	19%
Nedgroup Inv Equity R	0.15	30	36%	0.25	34	18%
STANLIB Equity R	0.12	39	7%	0.25	35	18%
Interneuron Capital Equity	0.12	38	13%	0.24	36	12%
Metropolitan General Equity	0.15	31	35%	0.24	37	12%
Sasfin Equity	0.13	37	17%	0.24	38	11%
Maestro Equity A	0.14	36	23%	0.23	39	8%
STANLIB SA Equity R	0.11	40	0%	0.22	40	0%

The weighted adjustment for heteroscedasticity improves the accuracy and appropriateness of this commonly applied ratio and other similar ratios. It also improves the ratio's power by improving its ability to detect significant performance. Equation 2.19 is the t-test pertaining to excess performance. In order to convert the ratios in Table 5.2 (Equation 2.18) to a t statistic, we need only multiply by the square root of the number of observations in each case which in this table is 60 (60 months of data sampled). This exercise will show that the top thirty funds ranked by weighted Information Ratios have t statistics greater than two and are therefore significant at the 5% level. None of the un-weighted t-statistics derived from Table 5.2 are significant. This supports the greater efficacy of using a weighted time series of fund and benchmark returns to detect superior performance.

#### ***5.4.2.1 Weighting simple multi-period returns***

Correcting for heteroscedasticity in time series is essential for the appropriate application of OLS regression and t statistics. The argument can be extended to simple multi-period returns in a similar way: performances achieved in highly disperse markets should be given less importance in the assessment of fund management success than performances achieved in markets with low cross-sectional dispersion of asset returns. Analogously, a golfer's position on the leader board on a windy day is a murkier measure of their skill than their relative success on a clear day.

Since the scale of returns is pertinent in an absolute way, the weighting methodology can be adjusted to ensure that the weights add to one to preserve the scale. In other words, each period's return should be weighted by the inverse of the cross-sectional standard

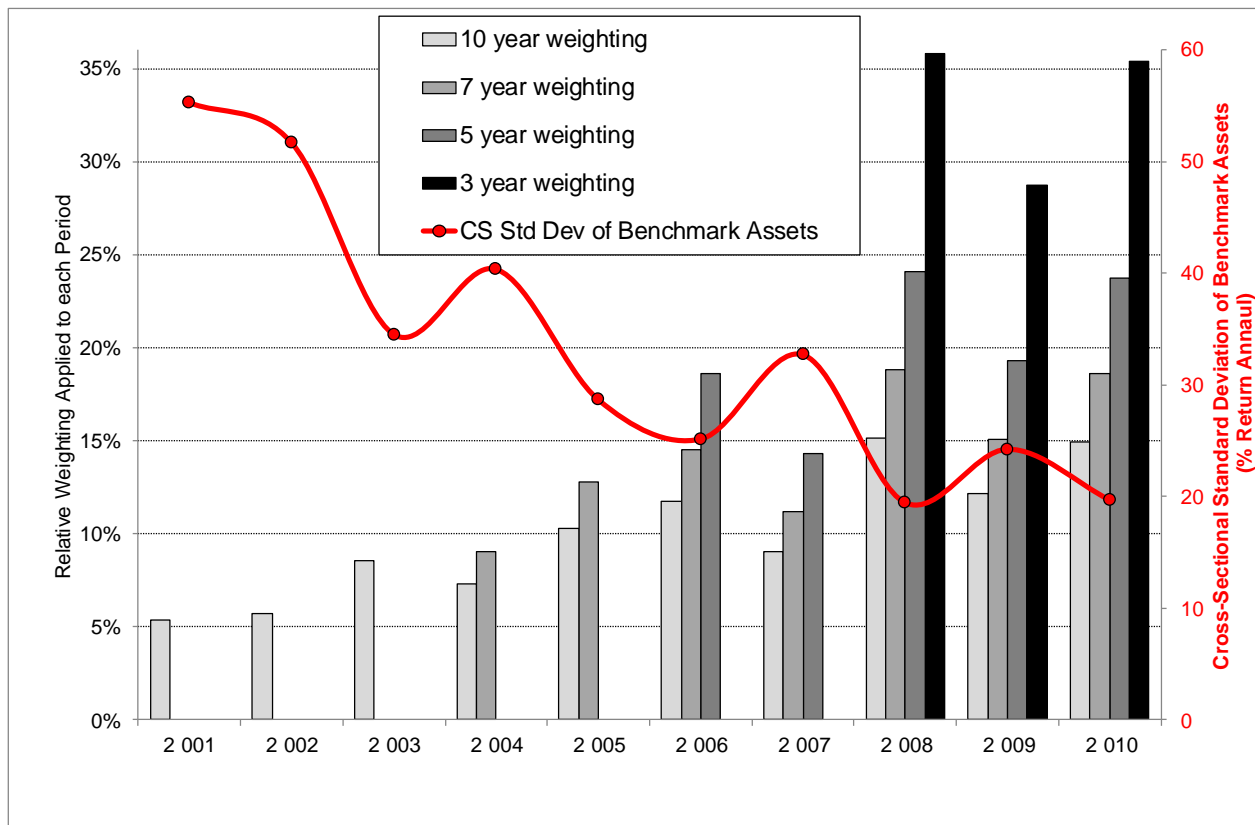
deviation of the benchmark as before but these weights can be adjusted so that they add to one over the full period of measurement (refer to Equation 5.4).

#### Equation 5.4: Weighted Returns

$$\left( \sum_{t=1}^T \frac{1}{s_{cs,t}} \right)^{-1} \sum_{t=1}^T \frac{R_{p,t}}{s_{cs,t}}$$

To illustrate, Figure 5.5 shows the weighting scheme that would apply to an assessment of annual fund returns over various investment intervals. As a reference, the annual cross-sectional standard deviation of benchmark assets is plotted on the secondary axis. Charted in this way it is easy to see the inverse and relative relationship these weightings have with the cross-sectional dispersion of the market.

For example, a three-year average annual return metric for the most recent calendar years on the ALSI would be weighted in such a way that the performance of 2008 and 2010 would contribute more to the average than that of 2009. In this way, performance achieved in the relatively high dispersion on the ALSI in 2009 is slightly discounted to account for its less significant indication of manager skill.



**Figure 5.5: Weightings to adjust the annual performances of General Equity funds over various investment horizons**

## 5.5 CONCLUSIONS

The dispersion in managed fund returns is closely related to the dispersion of the underlying securities in which they invest (i.e. the benchmark securities). This dispersion in fund returns and security returns varies substantially over time, resulting in heteroscedastic time series of returns in both. The ex post performance of competing portfolio managers should therefore be corrected for the varying cross-sectional risk of their investment environment in order to more accurately and fairly assess the extent of their success and skill. This is particularly true when using metrics related to regression or t statistics such as the Sharpe or Information Ratios as all these methods assume homoscedasticity in the series to which they are applied.

The weighted adjustments to performance time series proposed here allow manager performances to be compared on a more level playing field by considering the heteroscedasticity inherent in their investment environment during the history of the portfolio's performance. Returns delivered in periods of high dispersion in the benchmark assets are weighted less than returns earned in periods of low benchmark dispersion allowing for more efficient and appropriate detection of manager skill. The correction is simultaneously measurable, objective and free of sampling differences that might arise from estimating the source of heteroscedasticity directly from the sample of fund performances.



**Table 5.3: Breusch-Pagan regression results before and after adjusting for benchmark dispersion**

		Breusch-Pagan regression <sup>44</sup> using the residuals of Equation 5.1			Breusch-Pagan regression using the residuals of Equation 5.3		
Fund Name	# of months	R <sup>2</sup>	F-stat	P-value	R <sup>2</sup>	F-stat	P-value
Inv Solutions MM Equity A	112	0.22	30.2	0.000	0.07	8.3	0.005
BJM Core Equity B1	39	0.01	0.3	0.591	0.01	0.5	0.466
Aylett Equity A3	55	0.23	16.2	0.000	0.07	3.9	0.054
Personal Trust SA Equity	18	0.27	5.8	0.028	0.01	0.2	0.695
STANLIB MM Equity A1	121	0.21	31.2	0.000	0.06	7.5	0.007
Maestro Equity A	67	0.15	11.6	0.001	0.01	0.7	0.394
STANLIB Equity R	121	0.18	26.3	0.000	0.01	0.9	0.352
Discovery Equity	38	0.38	22.4	0.000	0.31	16.2	0.000
Metropolitan Multi-Manager Equity	41	0.05	2.0	0.170	0.00	0.1	0.722
Coronation Equity R	121	0.13	17.7	0.000	0.02	2.3	0.131
Verso LT SA Equity R	28	0.04	1.0	0.337	0.01	0.3	0.586
NFB Equity A2	28	0.00	0.0	0.843	0.11	3.2	0.085
NeFG Equity	30	0.06	1.7	0.203	0.00	0.1	0.773
Kagiso Equity Alpha	81	0.14	13.0	0.001	0.00	0.2	0.620
MI-PLAN IP Beta Equity B2	60	0.24	18.4	0.000	0.05	3.0	0.089
Prescient Eqty Active Quant A1	50	0.19	11.2	0.002	0.02	1.0	0.319
Huysamer Equity A1	55	0.15	9.5	0.003	0.03	1.4	0.248
Lion of Africa General Equity	38	0.27	13.1	0.001	0.04	1.6	0.210
Harvard House General Equity	52	0.24	16.2	0.000	0.00	0.1	0.793
ABSA Select Equity	83	0.10	9.1	0.003	0.02	1.5	0.224
Old Mutual Growth R	121	0.34	61.5	0.000	0.01	1.8	0.182
Old Mutual Top Companies R	121	0.25	40.5	0.000	0.01	1.2	0.284
Prudential Equity A	121	0.11	15.4	0.000	0.03	3.9	0.051
Nedgroup Inv Rainmaker A	121	0.17	24.5	0.000	0.01	1.4	0.232
Allan Gray Equity A	121	0.08	10.7	0.001	0.00	0.1	0.770
Indequity Technical	73	0.31	32.0	0.000	0.02	1.1	0.294
Analytics Managed Equity A	113	0.19	26.1	0.000	0.07	8.8	0.004
Nedgroup Inv Quants Core Eq	121	0.14	18.9	0.000	0.02	2.6	0.112
STANLIB Nationbuilder A	48	0.08	4.2	0.047	0.00	0.1	0.812
Element Earth Equity A	111	0.18	23.9	0.000	0.01	1.3	0.260
FNB Growth	121	0.14	19.9	0.000	0.00	0.4	0.514
Community Growth Equity	121	0.10	13.6	0.000	0.00	0.2	0.643
SIM General Equity R	121	0.14	18.9	0.000	0.03	4.2	0.044
ABSA General R	121	0.09	12.3	0.001	0.03	3.1	0.080
27Four Active Equity A1	28	0.01	0.2	0.680	0.03	0.7	0.417
Imara Equity	32	0.20	7.7	0.010	0.07	2.2	0.147
Afena Equity A1	32	0.13	4.4	0.043	0.04	1.1	0.295

<sup>44</sup> As per Equation 5.2.

		Breusch-Pagan regression <sup>45</sup> using the residuals of Equation 5.1			Breusch-Pagan regression using the residuals of Equation 5.3		
Fund Name	# of months	R <sup>2</sup>	F-stat	P-value	R <sup>2</sup>	F-stat	P-value
Interneuron Capital Equity	82	0.16	14.8	0.000	0.02	1.5	0.221
Coris Capital General Equity	121	0.18	26.3	0.000	0.06	7.4	0.007
STANLIB SA Equity R	121	0.19	28.4	0.000	0.01	1.2	0.270
Valugro General Equity	70	0.08	6.1	0.016	0.00	0.0	0.878
Hermes Equity A	68	0.09	6.3	0.014	0.00	0.0	0.925
Foord Equity R	101	0.18	22.0	0.000	0.01	0.7	0.412
Cannon Equity A	67	0.28	25.5	0.000	0.04	2.5	0.117
Old Mutual Investors R	121	0.12	16.8	0.000	0.02	2.4	0.123
Sasfin Equity	63	0.14	9.7	0.003	0.01	0.4	0.506
SIM Top Choice Equity A1	53	0.12	7.1	0.010	0.03	1.6	0.212
Investec Active Quants A	70	0.06	4.2	0.045	0.00	0.0	0.919
RMB Equity R	121	0.12	16.7	0.000	0.01	0.7	0.405
Old Mutual Active Quant Equity	121	0.11	14.5	0.000	0.01	1.1	0.300
Metropolitan General Equity	121	0.17	23.8	0.000	0.02	2.6	0.110
Investec Equity R	121	0.08	10.9	0.001	0.02	2.0	0.158
PSG Equity A	121	0.38	73.3	0.000	0.06	7.7	0.006
Sasfin Twenty Ten	63	0.14	9.7	0.003	0.00	0.1	0.749
Gryphon All Share Tracker	121	0.26	42.4	0.000	0.04	4.5	0.036
Nedgroup Inv Equity R	121	0.14	19.3	0.000	0.04	5.1	0.026
Efficient General Equity	28	0.11	3.1	0.089	0.04	1.0	0.332
Alphen Equity Builder	24	0.08	1.8	0.192	0.00	0.1	0.815
Plexus RAFI® Enhanced SA Strategy	41	0.25	12.9	0.001	0.11	5.0	0.031
Oasis General Equity	112	0.07	8.8	0.004	0.00	0.4	0.542

---

<sup>45</sup> As per Equation 5.2.

## **Chapter 6 : Constraints on investment weights: What fund sponsors in concentrated equity markets such as South Africa need to know**

### **6.1 SUMMARY**

This chapter demonstrates the importance of setting risk budgets and constraints mindful of the nature of the chosen benchmark and the investment environment. The optimal distribution of investment weights in each security is estimated in the context of standard portfolio construction techniques and typical South African equity benchmarks and market conditions. These distributions provide guidance to mandate authors who are considering allowing limited shorting in their net long portfolios as to the amount of gearing that is likely to be required. These estimates also show authors of long-only mandates the circumstances and assets for which their restrictions are materially binding.

### **6.2 INTRODUCTION**

This chapter uses the derivations published in Clarke *et al.* (2008) to determine the distribution of the optimal investment weights under various market conditions. While fund managers are concerned with the efficient implementation of their investment view into an investment portfolio, this analysis is particularly intended for the use of mandate authors who are concerned with finding the reasonable boundaries to fund manager activities. The methodology and empirical analysis in this section provide insights as to the likely distribution of optimal investment weights across various investment forecasts and subject to benchmark choice and risk budget. This chapter highlights the importance of the choice of benchmark and risk budget in determining the distribution of optimal

investment weights in the portfolio and, by implication, the extent of the sub-optimality of certain security-specific restrictions in the light of these two choices.

The analysis extends to determine the likely optimal fund gearing or short extension under various market conditions and risk budgets for those mandate authors who are considering relaxing the long-only constraint and concludes thereafter.

### **6.3 ACTIVE WEIGHTS AND THE RISK BUDGET**

When setting mandates, the fund sponsors must consider the necessary long-term objectives and constraints independently of a particular investment view that varies in the short term. In Chapter 3, we examined the boundaries of the active opportunity set given particular benchmarks and constraints. In this chapter, we attempt to quantify likely active positions in each security in the context of standard portfolio construction techniques and particular benchmarks. Using this analysis, we can estimate the extent of the short positions that will likely be required by an optimal, actively managed portfolio. These estimates provide some guidance to fund sponsors who are considering allowing limited shorting in their net long portfolios but are also an indication to fund sponsors who require long-only mandates as to the circumstances under which their restrictions are materially binding.

### 6.3.1 Optimal distributions of security weights

#### 6.3.1.1 *Unconstrained optimal portfolio weights*

Our starting point is the unconstrained<sup>46</sup> portfolio optimisation problem in an active management framework, namely to maximise the forecasted active return of the portfolio, while achieving a particular active risk target and ensuring that the portfolio is self-funding (refer Equation 2.6). This problem has a well-known, unique solution for any given active risk target (refer Equation 2.7).

Equation 2.7 shows that, when the portfolio is unconstrained, the size and sign of the active weight of any particular asset is directly related to the size and sign of the forecasted active return for that asset. The target active risk magnifies the extent of this bet or active weight while the risk in the denominator has the opposite effect. Thus a positive excess-of-benchmark return expectation (relative to the other forecasts) for a particular security would lead a rational, unconstrained and optimal fund manager to a positive active position in that same stock. The greater the target active risk (i.e. the lower the risk-aversion) of the investor, the greater the active positions in their portfolio will be. Conversely, the extent of this active position is reduced by the uncertainty of the asset's prospects.

---

<sup>46</sup> This optimisation requires self-financing and is sometimes solved for a particular active risk limit. Although both of these are constraints on the optimisation, the optimisation problem is generally referred to as an “unconstrained” optimisation on account of there being no constraints placed on individual weights in the portfolio.

### 6.3.1.2 Optimal short weights

Sorensen, Hua and Quian (2007) demonstrated how Equation 2.7 could be rewritten to determine the condition under which any particular stock will be held short in an optimal unconstrained fund i.e. the conditions under which  $w_a < -w_b$ .

#### Equation 6.1: Condition for a short position in an unconstrained optimal fund

$$\alpha < -\frac{w_b}{\sigma_A} \frac{\sqrt{\alpha^T \Sigma^{-1} \alpha}}{\Sigma^{-1}}$$

The Sorensen *et al.* (2007) derivation in Equation 6.1 yields several insights. Firstly, the optimal unconstrained portfolio is more likely to have a short position in a security when the security itself has a low weighting in the benchmark (small  $w_b$ ). The implication being that, irrespective of the fund manager's investment view at any given time, short extension fund administrators should be more concerned with the ability to short smaller securities when preparing prime broking arrangements and scrip lending agreements than the larger, more liquid securities. This is particularly true of small stocks with higher residual risks.

The second important insight yielded by Equation 6.1 is that the likelihood of an optimal short position in any security is increased across the board by higher risk budgets i.e. higher target active risk. The implication for fund sponsors being that, with greater active risk expectations<sup>47</sup> must come a greater loosening either of the typical constraints on fund managers or an increasingly sub-optimal, asymmetric portfolio construction setting.

---

<sup>47</sup>Presuming that the investment universe is not expanded in order to achieve greater active risk.

The active weight (and the potentially short holding) in any security is therefore not purely a function of the manager's forecast at any particular time and the risk characteristics of the assets under consideration, but is also dependent on the risk budget, the benchmark composition and each security's weight in the benchmark. Fund sponsors, by way of their choice of benchmark and their setting of the fund's risk objectives, therefore directly affect the distribution of likely weights assigned to assets during the portfolio construction process.

Considering then the sponsor's role in the portfolio construction process, the mandate authors need to be mindful of the distribution of active weights in the portfolio implied by the choice of benchmark and risk budget. To this end, an understanding of the distribution of likely security weightings in the portfolio is necessary.

### **6.3.1.3 Distribution of optimal unconstrained active portfolio weights**

Grinold (1994) proposed the "alpha generation" formula (refer Equation 2.8) which Clarke *et al.* (2006) generalised for a full covariance matrix in the following form:

#### **Equation 6.2: Alpha generation a la Clarke *et al.* (2006)**

$$\alpha = IC \Sigma^{-\frac{1}{2}} S$$

where  $S$  is an  $n \times 1$  vector of randomised standard normal scores.

Equation 6.2 presents forecasted excess returns as generated by a random normal process, scaled by skill ( $IC$ ) and risk ( $\Sigma$ ). Clarke *et al.* (2008) and Sorensen *et al.* (2007)

argue that, if forecast excess returns follow a random process, the distribution of optimal active weights that result from these forecasts can be derived accordingly. Using multiple simulations of the score ( $S$ ) in combination with their current risk model ( $\Sigma$ ) and risk budget ( $\sigma_A$ ) and solving for the unconstrained active portfolio, it is possible for mandate authors to derive distributions of active weights (and, by extension, the investment weights) for each asset in the investment universe. These simulated distributions of asset weights, consistent with a range of likely forecasts and the risk model, can then provide sound justification for various weight constraints that are appropriate for each asset and across changing investment views. This process can also inform the likelihood and extent of short selling optimally required in each stock thereby informing mandated constraints on short selling and preparing prime broking requirements in anticipation of future shorting requirements.

### 6.3.3 Distribution of active weights under simplified conditions

To answer the question of how much gearing/short extension a portfolio might need to optimise its use of forecast returns, Clarke *et al.* (2008) used a simplification to derive an analytical description of the relationships between the various parameters of fund management and the probability distribution of active weights. The authors used a simplified two-parameter variance-covariance matrix ( $\Sigma$ ): all individual asset variances are assumed to be equal ( $\sigma$ ), and all pairwise correlations have only one value, ( $\rho$ ). Under these assumptions, the authors combine Equation 2.7 and Equation 6.2 to show that the unconstrained optimal active weights are distributed normal with a mean of zero and a variance that is proportional to the active risk of the portfolio (Equation 6.3).



**Equation 6.3: Optimal unconstrained active weight distribution under the two-parameter covariance assumption**

$$w_a \sim N\left(0, \frac{\sigma_A}{\sqrt{N}} \frac{1}{\sigma \sqrt{1-\rho}}\right)$$

where

$\sigma$  is the standard deviation of returns of each security; and

$\rho$  is the correlation of the returns of each pair of securities.

This derivation in Equation 6.3 relates the required scope for active positions to both market and mandate conditions. Increasing volatility and decreasing correlation (i.e. increasing cross-sectional variation) result in a narrower distribution of active weights and a lower probability of needing short positions to achieve a particular active risk target in an optimal way.

Furthermore, Equation 6.3 demonstrates the role that the mandate author plays in the distribution of active weights by way of their choice of risk budget and benchmark. All things being equal, a wider distribution of active weights in each security is required with greater active risk targets ( $\sigma_A$ ). This increase in the spread of active weights increases exponentially by a reduced investment universe (smaller  $n$ ). These two insights alone confirm that funds managed on a smaller asset universe or benchmark will likely be more aggressive in their individual active weights per asset than funds managed in a more diverse universe if they are required to deliver the same active risk. Furthermore, the wider the distribution of active weights, the more likely short positions in the smaller stocks will be required in the optimal construction of a portfolio and, by implication, the more materially binding the long-only constraint will be on the portfolio's construction.

The insights gained from Equation 6.3 demonstrate the importance of setting risk budgets and constraints mindful of the nature of the chosen benchmark and the changing cross-sectional variation in the investment environment.

### **6.3.3.1 Empirical demonstration using the ALSI and ALSI 40**

Using the simplification described in 6.3.3, the following section illustrates the distribution of optimal security weights using an ALSI and an ALSI 40 benchmark. Figure 6.1 and Figure 6.2 depict the 95% confidence interval for the optimal unconstrained holdings in a selection of the ALSI constituents under a particular set of market conditions and a target active risk of 4% p.a. The circles represent the neutral position or benchmark holding which, following from Equation 6.3, is also the most likely position. Figure 6.1 and Figure 6.2 represent market conditions where the cross-sectional dispersion is low (35% p.a.) and high (70% p.a.) respectively, representing two historical extremes in the realised cross-sectional dispersion of the ALSI<sup>48</sup>. Figure 6.3 and Figure 6.4 represent the same scenarios but use the ALSI 40 as a benchmark instead.

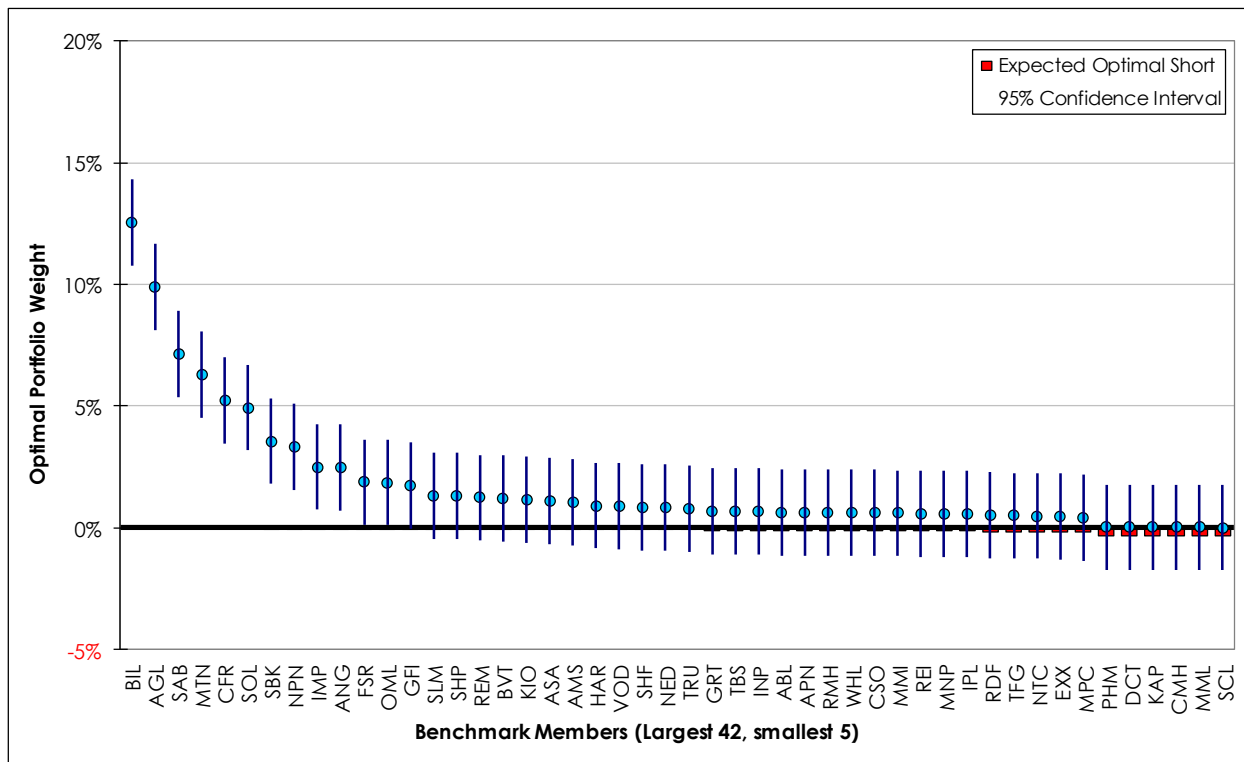
One of the most striking features of these figures is the greater short positions covered by the stocks with a smaller weight in the benchmark. The implication for fund managers is that, when prime broking is sought for short extension products, it is the smaller stocks that will most often be required to be held short. Conversely, trustees with concerns regarding the wholesale lifting of short sale constraints should note that not all the stocks in the

---

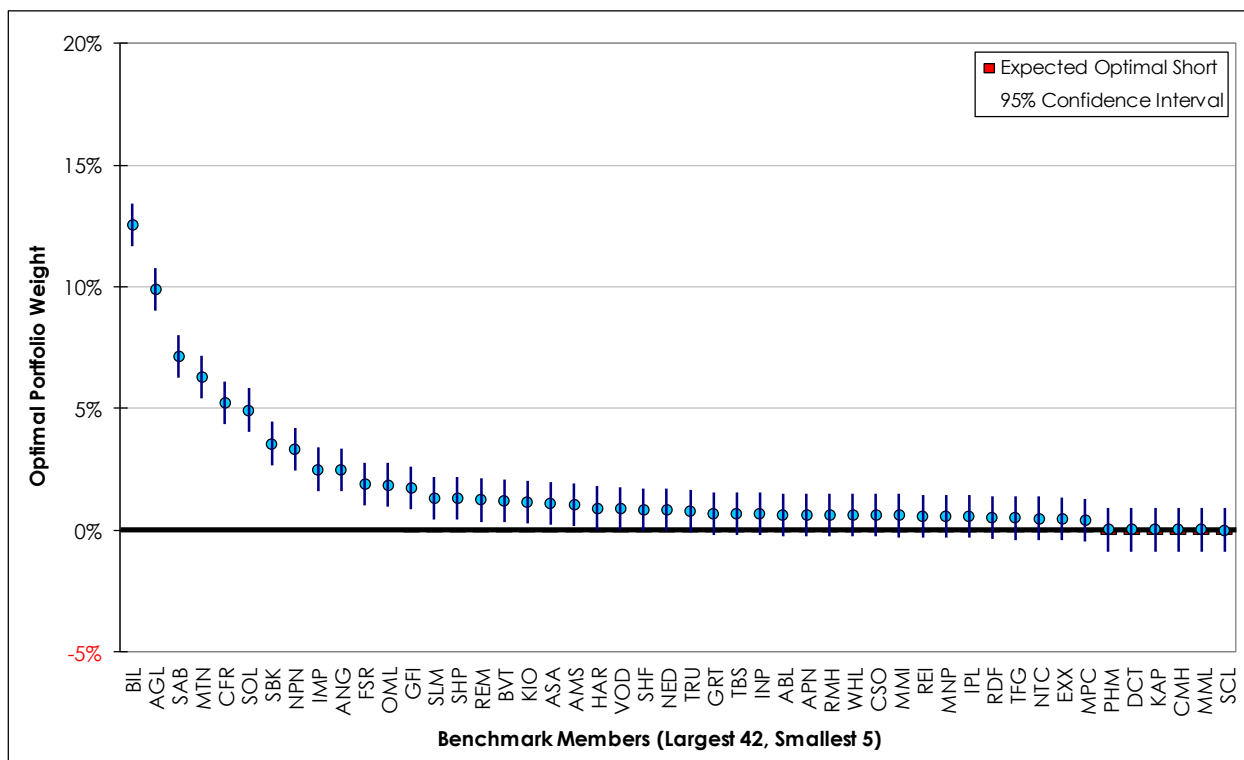
<sup>48</sup> Refer to Section 4.3 for the empirical record of the cross-sectional dispersion on the ALSI. This section shows the weighted cross-sectional standard deviation varying between roughly 10% and 20% in monthly percentage returns, 35% and 70% in annual terms.

investment universe require the long-only constraint to be removed. Figure 6.1 and Figure 6.2 show that the top twelve stocks, comprising 60% of the weight of the investment universe, may be constrained to be held long only in an ALSI fund with a fairly aggressive risk budget without material detriment to the optimality of the investment, even in a low dispersion market. In fact, to allow short selling in these larger securities, in the light of the evidence presented here, could be considered reckless. The same is true of a fund that is benchmarked against the smaller ALSI 40 index (refer to Figure 6.3 and Figure 6.4): 60% of the weight of this index is comprised of the largest eight stocks which are unlikely to be required in short positions by an optimal portfolio.

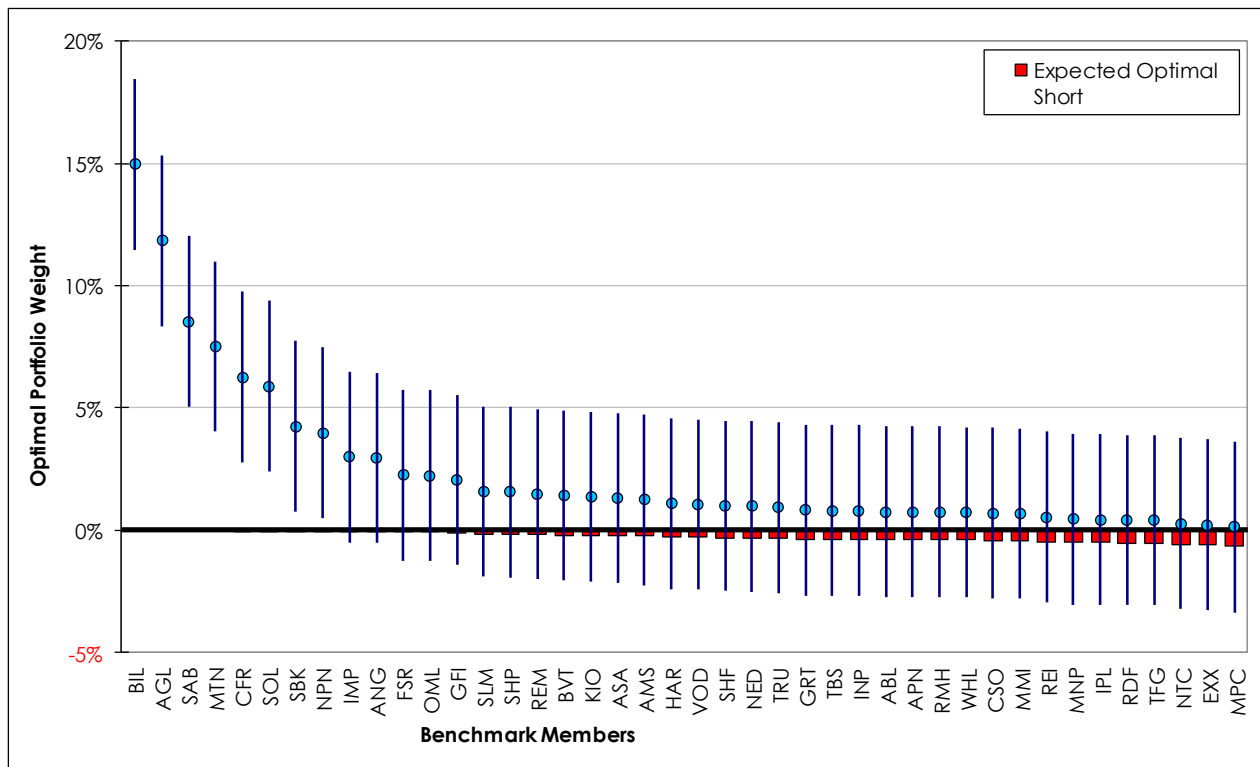
Another feature of these figures is the difference in the spread of the distribution of optimal weights across assets under different prevailing cross-sectional variance. When the cross-sectional dispersion is high (as in Figure 6.2 and Figure 6.4), the risk budget is more easily attained without substantial shorting. Under these conditions, a long-only manager is not as handicapped as they would be in a low dispersion market (as in Figure 6.1 and Figure 6.3) where optimal conditions will likely require short positions across a wider variety of assets. This also shows that a rigid long-only constraint across all assets will vary in its detrimental impact on the portfolio as the dispersion in the market's opportunities varies.



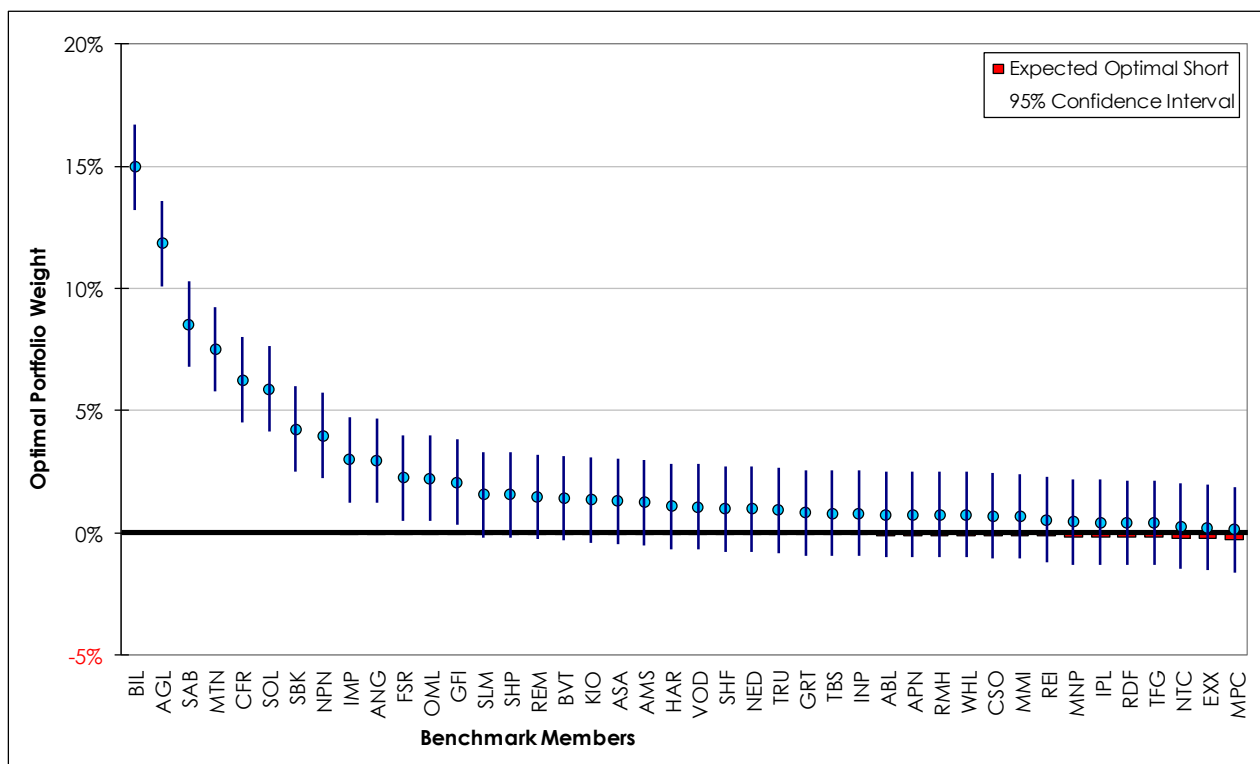
**Figure 6.1: 95% Confidence intervals for optimal unconstrained holdings in each benchmark security ( $\sigma\sqrt{1-\rho} = 0.35$ , active risk=4% p.a., benchmark: ALSI)**



**Figure 6.2: 95% Confidence intervals for optimal unconstrained holdings in each benchmark security ( $\sigma\sqrt{1-\rho} = 0.75$ , active risk=4% p.a., benchmark: ALSI)**



**Figure 6.3: 95% Confidence intervals for optimal unconstrained holdings in each benchmark security ( $\sigma\sqrt{1-\rho} = 0.35$ , active risk=4% p.a., benchmark: ALSI 40)**



**Figure 6.4: 95% Confidence intervals for optimal unconstrained holdings in each benchmark security ( $\sigma\sqrt{1-\rho} = 0.75$ , active risk=4% p.a., benchmark: ALSI 40)**

Although the analysis in this section makes use of a simplification that is not cognisant of the manager's risk model, it provides a good understanding of the inappropriateness of one-size-fits-all mandated constraints on asset weights that is relatively easy to implement. Mandate authors could make use of the simplified analytical relationships presented here to quantify the distribution of optimal weights implied by their choice of benchmark and risk budget. Using various cross-sectional dispersion scenarios and their chosen benchmark's composition, mandate authors could observe the extent to which their imposed constraints on asset weights are likely to be binding.

#### **6.3.4 Simulated active weight distributions**

Simulation can be used to extend this analysis beyond the simplification of a two-parameter covariance matrix. The complex relationships between the various securities in an investment universe are usually estimated and modelled with a risk model. Such a model is used in both the portfolio construction process (ex ante) and the performance monitoring and reporting (ex post) and can be incorporated in a simulation exercise to derive distributions of active weights that are more consistent with these processes.

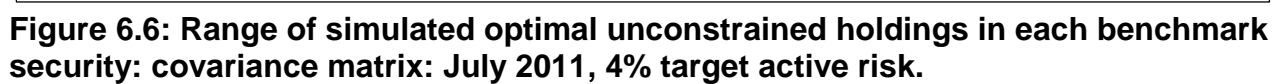
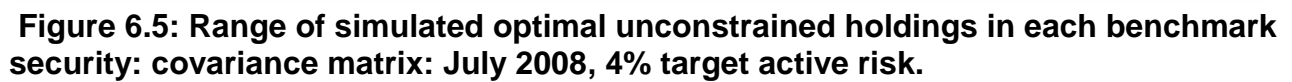
The starting point of such a simulation is the alpha-generation model as per Equation 6.2. Empirical research such as Scherer and Xu (2007) and Clarke *et al.* (2006) have used this relationship to simulate a set of forecasts ("alphas") in order to illustrate elements of portfolio construction. In this analysis, multiple forecasts are simulated in order to examine the distribution of investment weights across these forecasts. To begin, 80 000 random numbers were generated from the random normal distribution to represent 500 investment views: i.e. 500 cross-sectional scores ( $S_i$ ) across the top 160 shares on the

ALSI. These scores were transformed into “alphas” or active forecasts using an IC<sup>49</sup> of 0.1 a la Clarke *et al.* (2006) and a covariance matrix sampled from MSCI Barra Aegis South African risk model using the relationship in Equation 6.2. Next, these alphas were transformed into optimal unconstrained portfolio weights using the relationship described in Equation 2.7 for a variety of target active risks and using the same covariance matrix.

In order to see the effect of different cross-sectional risk conditions on the distribution of portfolio weights, two covariance matrices ( $\Sigma$ ) were sampled historically from MSCI Barra Aegis South African risk model: one sampled during a period of relatively high cross-sectional variance (July 2008) and one from a period of relatively low cross-sectional variance (July 2011). The same scores ( $S$ ) were applied to each of the covariance matrices in order to generate alphas according to Equation 6.2 and portfolios according to Equation 2.7 that are representative of the same investment views but different market conditions.

---

<sup>49</sup> The choice of IC in this context is arbitrary. The IC scales the alphas and therefore also the active weights in the portfolios. In this study the active weights are compared across varying active risks and covariance matrices: as long as a consistent and realistic IC is used throughout, the size of the IC is not important.





The distributions of weights in each security across 500 simulations are shown in Figure 6.5 and Figure 6.6. Both of these figures represent portfolios with 4% active risk: the former using a covariance matrix representing a generally high cross-sectional dispersion (July 2008) and the latter using a covariance matrix representing a generally low cross-sectional dispersion (July 2011). The sensitivities of the simulation study are consistent with the analysis in sections 6.3.3: a) the larger the weight of the stock in the index, the less likely the stock is to be held short in an unconstrained optimal portfolio and b) the less cross-sectional dispersion among securities, the wider the distribution of security weights.

### **6.3.5 Fund level constraints on gearing and the risk budget**

The previous section showed that smaller stocks are more likely to be held in short positions than larger ones. In this section, we consider the optimal aggregate short position for the portfolio as a whole. Both the simplified, analytical derivation presented in Section 6.3.3 and the simulations generated in Section 6.3.4 can be aggregated to estimate the optimal likely fund gearing. Mandate authors can use these aggregate estimates to determine the amount of short extension that will likely be required under particular covariance assumptions and for a variety of envisaged active risk targets.

#### **6.3.5.1 Two parameter covariance matrix simplification**

Using the distribution of asset weights under the two-parameter covariance assumption (Equation 6.3), Clarke *et al.* (2008) derived the most likely short position of each asset, conditional on the asset being held short at all (Equation 6.4). These likely short positions can be calculated for each benchmark security and aggregated to provide an estimate of the overall short position that is most likely for an optimal portfolio.

**Equation 6.4: Expected short selling weight conditional on short sales being required (Clarke *et al.*, 2008)**

$$E(w_i | w_i < 0) = cf\left(-\frac{w_{b,i}}{c}\right) - w_{b,i}P\left(z < -\frac{w_{b,i}}{c}\right)$$

where

$$c = \frac{\sigma_A}{\sqrt{N}} \frac{1}{\sigma \sqrt{1-\rho}}; \text{ and}$$

$f(\cdot)$  is the normal density function.

To illustrate, the thick negative bars in Figure 6.1, Figure 6.2, Figure 6.3 and Figure 6.4 represent the conditional expected short position in each of these stocks for each benchmark and under each of the two cross-section dispersion assumptions. In other words, if each security is to be held short, the bars show the extent of the short position we expect on average given these market and investment conditions.

Notice once again how the expected short positions on larger assets is zero but that the expected short positions increase with decreasing asset weight. In an unconstrained ALSI fund (as represented by Figure 6.1 and Figure 6.2) there are a large number of small expected short positions spread among 150 to 125 smaller stocks whereas an optimal unconstrained ALSI 40 fund (as represented by Figure 6.3 and Figure 6.4) would likely require larger short positions over a smaller number of assets. The expected short positions increase across the board with decreasing cross-sectional dispersion.

Adding up the expected short positions per security provides us with the expected level of shorting in the portfolio as a whole. This in turn allows authors of short extension mandates to form expectations as to the level of gearing that will likely be required over

various investment views, consistent with the choice of benchmark and risk budget. By implication, again, mandate authors for long-only funds can use this as a metric for the sub-optimality of their choice of risk budget and benchmark: the more gearing is required for an optimal unconstrained fund under the same circumstances, the more inappropriate the choice of benchmark and risk budget is for a long-only fund.

Table 6.1 and Table 6.2 provide some examples of these estimates under a variety of market conditions and target active risk levels for both an ALSI and an ALSI 40 active fund respectively. Under cross-sectional volatility conditions (i.e.  $\sigma\sqrt{1-\rho}$ ) that vary from 0.2 to 0.7 per annum, and risk budgets that vary from enhanced equity funds ( $\sigma_A=0.5\%$ ) to more aggressive active funds ( $\sigma_A=4\%$ ), these tables describe the optimal, unconstrained, likely level of gearing in the fund as a consequence of the likely short sales. For example, in a market where the cross-sectional volatility is 0.4 and the target active risk is 4%, the optimal unconstrained ALSI fund would expect 33.8% short sales (i.e. R33.80 of every R100 invested would be in short positions and R133.80 would be invested long). By contrast, the same requirements of an ALSI 40 fund would require a very modest average of only 2.28% gearing confirming the more efficient long-only implementation of these requirements in an ALSI 40 than an ALSI investment universe.

**Table 6.1: Sum of expected short positions (%) in an unconstrained optimal active ALSI fund**

Target Active Risk		0.5%	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	6.0%	7.0%
Cross-sectional Volatility	0.20	5.0	13.5	23.3	33.8	44.7	55.9	67.3	78.9	90.5	102.3	126.1	150.0
	0.30	2.7	7.7	13.5	20.0	26.8	33.8	41.1	48.4	55.9	63.5	78.9	94.5
	0.40	1.7	5.0	9.1	13.5	18.3	23.3	28.5	33.8	39.2	44.7	55.9	67.3
	0.50	1.2	3.6	6.6	9.9	13.5	17.4	21.3	25.4	29.6	33.8	42.5	51.4
	0.60	0.9	2.7	5.0	7.7	10.5	13.5	16.7	20.0	23.3	26.8	33.8	41.1
	0.70	0.6	2.1	4.0	6.1	8.5	10.9	13.5	16.2	19.0	21.9	27.8	33.8

**Table 6.2: Sum of expected short positions (%) in an unconstrained optimal active ALSI 40 fund**

Target Active Risk		0.5%	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	6.0%	7.0%
Cross-sectional Volatility	0.20	0.06	0.41	1.16	2.28	3.71	5.36	7.19	9.16	11.25	13.42	18.00	22.81
	0.30	0.02	0.14	0.41	0.86	1.50	2.28	3.21	4.24	5.36	6.57	9.16	11.96
	0.40	0.01	0.06	0.19	0.41	0.73	1.16	1.68	2.28	2.96	3.71	5.36	7.19
	0.50	0.00	0.03	0.10	0.22	0.41	0.66	0.98	1.36	1.79	2.28	3.40	4.68
	0.60	0.00	0.02	0.06	0.14	0.25	0.41	0.61	0.86	1.16	1.50	2.28	3.21
	0.70	0.00	0.01	0.04	0.09	0.16	0.27	0.41	0.58	0.79	1.03	1.60	2.28

This analysis confirms the increased gearing allowance required for optimal funds with a) decreasing cross-sectional dispersion in the investment universe, b) increasing risk budgets and c) increasingly concentrated benchmarks. By implication, the detrimental effect of the long-only constraint on the active fund manager's ability to implement their investment views worsens under the same conditions.

## 6.4 CONCLUSIONS

Chapter 3 showed that the long-only constraint, when applied to benchmarks with a large number of small members, implies a substantial asymmetry in the opportunity set of active weights, particularly when the allowable active weights are larger. The simple metric derived in Chapter 3 showed a substantial improvement to the opportunity set when small collective short positions are permitted.

This chapter extends the analysis of Chapter 3 using the derivations published in Clarke *et al.* (2008). The analysis in this chapter uses a simplified, two-parameter risk matrix and two common South African equity benchmarks (ALSI and ALSI 40) to illustrate the importance of the choice of benchmark and risk budget in determining the distribution of optimal investment weights in the portfolio. The more concentrated the benchmark and the higher the active risk target, the wider the distribution of asset weights will be and the more binding the typical restrictions on asset weightings will be. Furthermore, constraints on short positions are more binding on assets with low-weightings in the benchmarks illustrating the asymmetric and sub-optimal effect of constraints applied equally to all assets in the investment universe.

The derivations and simulation techniques used in this chapter describe a way in which mandate authors could calculate the distribution of optimal investment weights to assist them in finding reasonable asset-level constraints that are consistent with their choice of benchmark and risk budget. By implication, mandate authors can use these techniques to determine the extent of the sub-optimality of certain security-specific restrictions, such as the long-only constraint.

The analysis presented in this chapter confirms the increased gearing allowance required for funds with decreasing cross-sectional dispersion in their market, increasing risk budgets and highly concentrated benchmarks with a large number of small securities. By implication, the detrimental effect of the long-only constraint on the active fund manager's ability to implement their investment views worsens under the same conditions. Using these techniques, mandate authors who are considering relaxing the long-only constraint can determine the likely optimal fund gearing or short extension under various market conditions and risk budgets. The evidence presented here suggests that small short extensions (less than 5% overall) may substantially improve the capacity for large cap managers (ALSI 40 benchmark) to achieve aggressive risk targets (4% active risk) in a more optimal way. Funds with broader, concentrated benchmarks such as the ALSI would require substantial short extension (to the order of 30%) to achieve the same risk budget efficiently<sup>50</sup>.

---

<sup>50</sup>This chapter did not address the issue of increased costs in short extension funds but the reader is cautioned that, aside from the additional costs of managing and implementing short positions, a 30% short extension fund incurs 160% of the transaction costs of a long-only fund.

## **Chapter 7 : Simulating the impacts of cross-sectional return dispersion and the long-only constraint on funds that are benchmarked against the ALSI**

### **7.1 INTRODUCTION**

The effect on the quality of investment portfolios both ex ante and ex post of the following pieces of the investment management problem have been explored in this thesis thus far:

a) concentration, b) long-only constraints, c) varying cross-sectional volatility and d) required active risk targets. In this chapter, simulation is used to demonstrate some of these effects empirically.

The investment decision is a response to the prevailing cross-sectional investment view of the entire investment universe available to a given portfolio manager at any given time subject to the market conditions and risk characteristics of the investment period. The investment portfolio is the manager's best response to these time-sensitive and usually proprietary forecasts. The research in this section measures the detrimental effects of the long-only requirement in a concentrated investment environment and the severity of these effects as a function of increasing target active risk and changing cross-sectional risk characteristics of the investment universe.

Historic data collection would be inadequate for this study for two reasons. Firstly, historic cross-sectional investment views are proprietary and are seldom stored historically since they are no longer useful to their owners (the portfolio constructors) once new views are formed. Secondly, the goal here is to observe the effects of various investment conditions

in general and so observing these effects under a particular investment view would under-represent the circumstances in which portfolio managers operate. Consequently, where analytical derivations are not possible, the effect on investment portfolios are illustrated here with simulated investment views combined with particular samples of historical risk characteristics.

Chapter 4 concerned the effect of varying cross-sectional risk on the investment decision and illustrated the inefficiency introduced when high active risks are required from portfolios in a low cross-sectional variance environment. Chapter 6 continued by demonstrating the distribution of likely security weights under various active risk and cross-sectional risk scenarios, hinting at the detrimental effects of restricting these distributions to weights above zero. In this section, the natures of portfolios that are constrained to be long only are compared to their unconstrained equivalent. The comparisons are made after aggregating portfolios generated from a variety of simulated investment views using the ALSI as the investment universe and benchmark. The aggregation across various views serves to isolate the effects of the long-only constraint under various active risk and cross-sectional risk scenarios, eliminating the influence of particular investment views.

In this way, the following question is addressed: how would two optimal fund managers, with the same investment view, have made different investment decisions under different conditions and constraints? The next section will outline the methodology in more detail.



## 7.2 DATA AND METHODOLOGY

### 7.2.1 Standard portfolio optimisation problem

The investment decision takes the form of a portfolio of investments, a manager's best response to a cross-sectional forecast of their entire allowable investment universe and the various risk characteristics of these investments subject to various constraints. The forecast at any given time is expressed as a vector of active returns or "alphas". These alphas must be translated into an investment portfolio that optimally orientates the size of the relative investment in each asset so that the payoff between the consequent risk and forecast return for the entire investment portfolio is satisfied. The portfolio construction problem is therefore one of translating a cross-sectional investment forecast into an investment portfolio having considered all constraints and limitations.

#### ***7.2.1.1 The unconstrained optimal investment portfolio***

The unconstrained<sup>51</sup> portfolio optimisation problem in a benchmark relative framework, seeks to maximise the forecasted active return of the portfolio of assets subject to a particular active risk target. For any given active risk target, there is a well-known, unique solution (refer Equation 2.7) to this problem.

Equation 2.7 shows that the size and sign of the active weight of any particular stock is directly related to the size and sign of the forecasted active return when the portfolio is

---

<sup>51</sup> Although there is clearly a self-financing constraint imposed on this optimisation problem, it is generally referred to as an "unconstrained" optimisation on account of there being no constraints placed on individual weights in the portfolio.

unconstrained. The target active risk magnifies the extent of this bet or active weight while the risk in the denominator has the opposite effect. Thus a positive excess-of-benchmark return expectation (relative to the other forecasts) for a particular security would lead a rational, unconstrained and optimal manager to a positive active position in that same stock. The greater the target active risk (i.e. the lower the risk-aversion) of the investor, the greater the active positions in their portfolio will be. Conversely, the extent of this active position is diminished by the uncertainty of the security's prospects.

### **7.2.1.2 Constrained optimisation**

In practice, portfolios are subjected to multiple constraints, the most typical of which are the self-financing constraint, neutrality constraints and long-only constraints. These can be represented as equality and inequality constraints.

#### ***Equality constraints***

Equality constraints require certain portfolio parameters to be equal to a particular value. These can be written in the following general form:

#### **Equation 7.1: Typical portfolio equality constraints**

$$Aw = 0 \text{ where } A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ s_1 & s_2 & \cdots & s_n \\ \beta_1 & \beta_2 & \cdots & \beta_n \end{bmatrix}$$

The first line of the  $k \times n$  matrix  $A$  is the self-financing constraint that requires the sum of the active weights to be zero. The remaining constraints require some attribute such as size, beta etc to be neutralised i.e. the portfolios net active investment in each attribute (s)

should be zero. These constraints are intended to neutralise the portfolio's exposure to (and return contribution from) macro-factors. In practice, these factor exposures can be achieved synthetically using financial engineering or by way of specific factor mandates. Constraining the portfolio manager in this way effectively mandates the manager to be a "stock-picker" only and not to deliberately take advantage factor effects such as size, value etc.

In this study, only the self-financing constraints will be applied so as not to clutter the results with a selection of factors or with factor estimates and their errors. This self-financing constraint will be applied in all cases and any further reference to "unconstrained" portfolios should be taken to mean that the portfolio has not been constrained in any way other than to require it to be self-financing<sup>52</sup>.

### ***The inequality constraints***

The long-only constraint is one of many inequality constraints. Standard for most pension fund investments, the long-only constraint requires the active weight in each asset to be at least as large as the negative of the benchmark weight: in other words, the total weight invested in each asset must not be less than zero.

---

<sup>52</sup> Grinold (1994) shows that the difference between an optimal portfolio with and without this constraint can be captured by a constant shift in the alpha vector. Particularly  $\alpha - \frac{1'\Omega^{-1}\alpha}{1'\Omega^{-1}\mathbf{1}}\mathbf{1}$  where  $\mathbf{1}$  is a vector of ones.

**Equation 7.2: The long only portfolio constraint**

$$w_a \geq -w_b$$

where  $w_b$  is the  $n \times 1$  vector of benchmark weightings in each asset.

In practice, many additional asset-specific constraints are expressed in the same inequality form. Minimum and/or maximum boundaries on the active weights and/or total weights of individual assets are one such example. Regulations such as South Africa's Regulation 28 and the fund's own investment rules require portfolios not to be too heavily invested in any one asset to avoid excessive and irresponsible concentration of the investment portfolio. One of the observable characteristics of interest to this study is the concentration of the optimal portfolios under various conditions. For this reason, no ad-hoc security specific constraints will be applied other than the long-only constraint so as not to interfere with the observation of concentration in this study. Of course, in practice, applying other inequality constraints will have a material influence on the results quoted here. However, the aim of this research is to find solutions for risk management and concentration that are informed by the forecasts and market conditions rather than ad hoc numerical limits to investment weights.

**7.2.2 Data****7.2.2.1 Benchmark and investment universe**

Throughout this study, the ALSI will be used as the benchmark and the members of the ALSI will comprise the investment universe. The ALSI represents the concentration and limitations of the South African equity manager. In the first phase of this analysis, the recent (month end July 2011) composition of the ALSI is used as a benchmark under two

different market risk scenarios. In the second phase of this analysis, the market conditions as at month end July 2011 are compared with the market risk conditions of the month end July 2008. These two snapshots in time represent relatively low (July 2011) and relatively high (July 2008) cross-sectional variance conditions on the ALSI (refer to Chapter 4 for the empirical evidence of the changing historical cross-sectional volatility on the ALSI). To be consistent, the composition of the ALSI as it was on each of those dates is applied as the benchmark. However, in order to compare the portfolios constructed under different circumstances for the same stock-specific scores, the ALSI was reweighted so that only the largest 160 stocks were included in the investment universe in each case. This amounted to the exclusion of two stocks with a collective weight in the index of 1.5bp in 2011 and the exclusion of four stocks with a collective weight in the index of 2.5bp in 2008.

#### **7.2.2.2 Alpha generation and the covariance matrix**

Grinold (1994) proposed the alpha generation formula which Clarke *et al.* (2006) generalised for a full covariance matrix (refer Equation 6.2). Empirical research such as Scherer *et al.* (2007) and Clarke *et al.* (2006) have used this relationship to simulate one set of forecasts (“alphas”) in order to illustrate elements of portfolio construction. The analysis in this chapter will make use of multiple simulations of cross-sectional investment forecasts to represent a wide variety of investment views on which manager’s would construct their investment portfolios. Specifically, 500 sets of cross-sectional alpha forecasts will be derived using 500 sets of simulated, standard normal scores ( $S$ ) across all

160 securities<sup>53</sup> and a single IC<sup>54</sup> of 0.1 a la Clarke *et al.* (2006). The covariance matrix used to generate these views (as per Equation 6.2) will be the same in each case as the covariance matrix used to construct each of the portfolios (refer Equation 2.7).

Four covariance matrices will be used: two simplified matrices and two sampled historically from MSCI Barra Aegis South African risk model. Each of these two pairs represents one market with high cross-sectional covariance and one market with low cross-sectional covariance. Following on the simplifications of Clarke *et al.* (2008) the first two covariance matrices are two-parameter covariance matrices. That is, each covariance matrix is comprised of the same variance for each stock and the same correlation for each pair of stocks.

The first two-parameter matrix has correlation of 0.3 and a standard deviation of 0.6 ( $\sigma\sqrt{1-\rho} = 0.50pa = 14.49\%pm$ ), representing a high cross-sectional variance market with a high carrying capacity for active risk taking. The second two-parameter matrix has a correlation of 0.6 and a standard deviation of 0.4 ( $\sigma\sqrt{1-\rho} = 0.25pa = 7.3\%pm$ ) representing more restrictive risk conditions and lower cross-sectional covariance. The choice of parameters are an attempt to represent two contrasting and relatively extreme representations of cross-sectional variance within the scale provided by the empirical evidence presented in Chapter 4 and represented in Figure 4.1.

---

<sup>53</sup> In other words, 500 times 160 random numbers were generated from a standard normal distribution to conduct this study.

<sup>54</sup> Although the IC in Clarke (2006) is set to 0.1, in the empirical illustration in this article, the ex post (realised) IC is less (0.068).

The two historically sampled covariance matrices were selected to represent similar contrasting market risk conditions: one matrix was sampled during a period of relatively high cross-sectional variance (July 2008) and one from a period of relatively low cross-sectional variance (July 2011)<sup>55</sup>.

In each case, the same covariance matrix will be used to a) generate alphas from the randomised scores ( $\mathcal{S}$ ) that represent the investment view and b) construct the optimal portfolios. Furthermore, the same scores ( $\mathcal{S}$ ) will be applied to each of these four covariance matrices in order to generate alphas and portfolios that differ only in respect of their constraints and market risk conditions. When applying the historical matrices, the differences in the construction of the benchmark will also come into play. As mentioned earlier, this influence is mitigated by a) using only the largest 160 stocks in the index and b) applying the scores to stocks of consistent rank. For example, the 121<sup>st</sup> the largest stock in July 2008 will be allocated the same score as the 121<sup>st</sup> largest stock in July 2011 in each simulation. In this way, the influence of the relative size of each stock comes into play but the name and characteristics of the stocks are irrelevant.

By constructing the analysis in this way, constrained portfolios across all simulated investment views ( $\mathcal{S}$ ) can be consistently compared against their unconstrained counterparts and these can in turn be compared across the four covariance matrices. The comparisons therefore isolate the differences in portfolio construction and market risk

---

<sup>55</sup> Refer to Section 4.3 for an empirical summary of historic cross-sectional variance in the South African Equity Market.

circumstances and, to some extent, neutralise the differences in the investment view with which these portfolios are constructed by analysing the cross-section of portfolios across 500 different simulations.

### **7.2.3 Bases of Comparison**

Optimally constructed portfolios will be compared in terms of various portfolio metrics or properties. The specifics of each metric are discussed in Section 7.3. Each metric represents an element of portfolio quality. The comparison will be between constrained and unconstrained optimal portfolios in order to illustrate the effects of the long-only constraint within the context of a concentrated investment universe: in this case, the ALSI. These comparisons are made across multiple simulated alpha forecasts in summary form i.e. the distribution of each metric described in Section 7.3 is represented by summary statistics across all the portfolios generated from the set of 500 simulations. In this way, the comparison is between the general distribution of a metric derived across all simulated investment views, with and without the long-only constraint.

Additionally, these comparisons will be made across a continuum of active risk targets and using four different covariance models. This allows us to quantify the deterioration of the effect of the long-only constraint under a) increasingly aggressive active risk mandates and b) market conditions that allow less capacity for risk taking (i.e. exhibit lower cross-sectional variation).



### 7.3 ANALYSIS

There are a number of different ways to measure the quality of a portfolio, each with a different perspective. This section describes the various metrics used in this research.

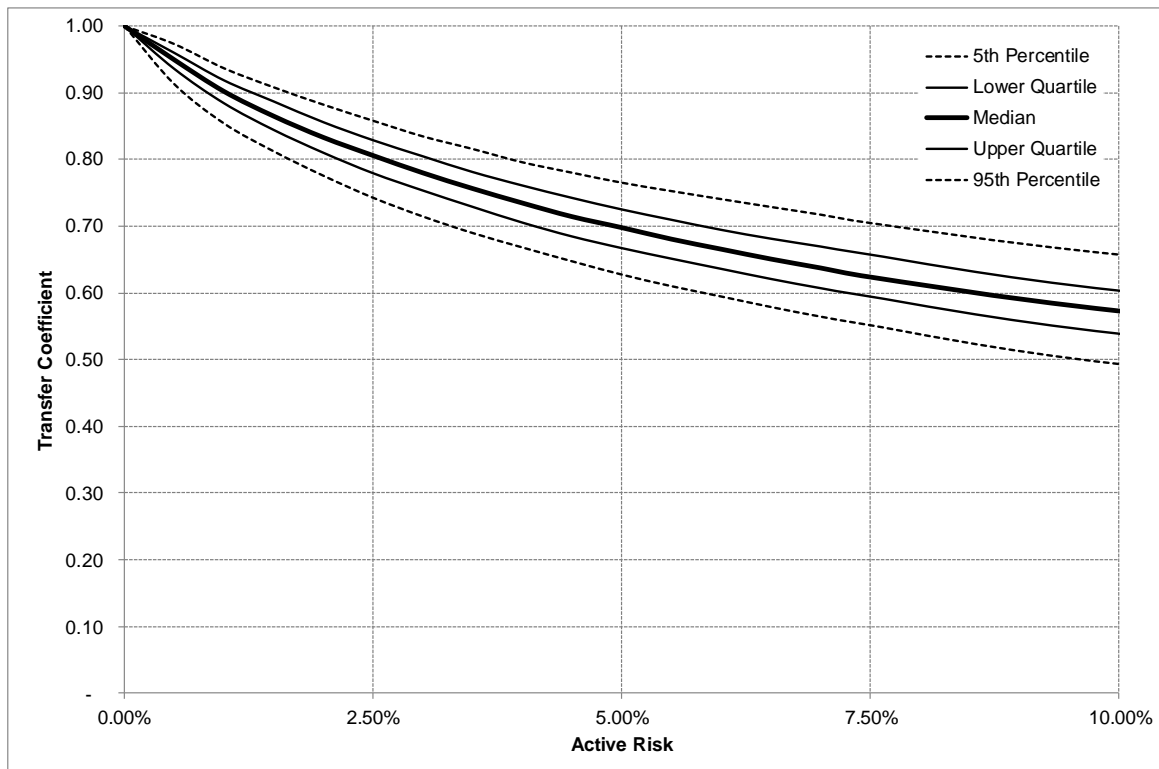
#### 7.3.1 Transfer coefficient

The transfer coefficient (TC) is a single, cross-sectional, ex ante measure of the consistency of a portfolio with the forecasts on which it was constructed. The TC of an optimal unconstrained portfolio is one, representing the perfect correlation between forecasts and investment weightings. When binding constraints are introduced into the portfolio construction process, the TC of the optimal constrained portfolio reduces.

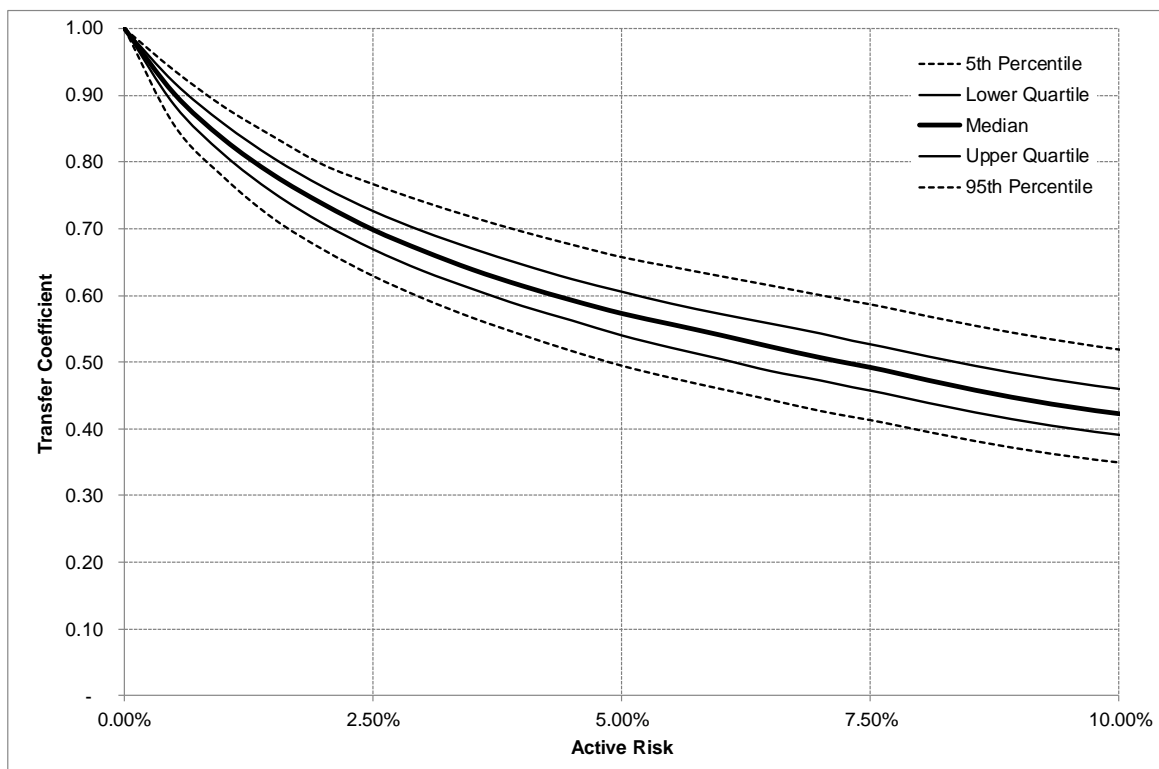
Under the simplifying assumption of a diagonal covariance matrix (a la Grinold (1994), the TC is defined by Equation 2.10 . In this chapter, we use the derivation of Clarke *et al.* (2006) of the same concept using a full covariance matrix (refer Equation 2.12). This definition holds for any portfolio, even a suboptimal one. However, in the context of this chapter comparisons will be made between the TC of the unconstrained optimal and the constrained optimal portfolios at each particular target active risk and across all simulations. The aim of the analysis is to illustrate a) the deterioration in the TC for long-only portfolios compared to unconstrained portfolios, b) the increased deterioration of TC with higher target active risk levels and c) the increased deterioration of TC when a risk model of lower cross-sectional variance is applied in portfolio construction.

The results of the application of the simplified, two-parameter covariance matrices are summarised in Figure 7.1 and Figure 7.2 for high and low cross-sectional dispersion respectively. The comparable results of applying historical covariance matrices for July 2008 (high cross-sectional covariance) and July 2011 (low cross-sectional covariance) are presented in Figure 7.3 and Figure 7.4. The 95<sup>th</sup> and 5<sup>th</sup> percentile, interquartile range and median of all the simulated portfolios are depicted in these figures in order to observe the distribution of the resulting transfer coefficients as a function of target active risk.

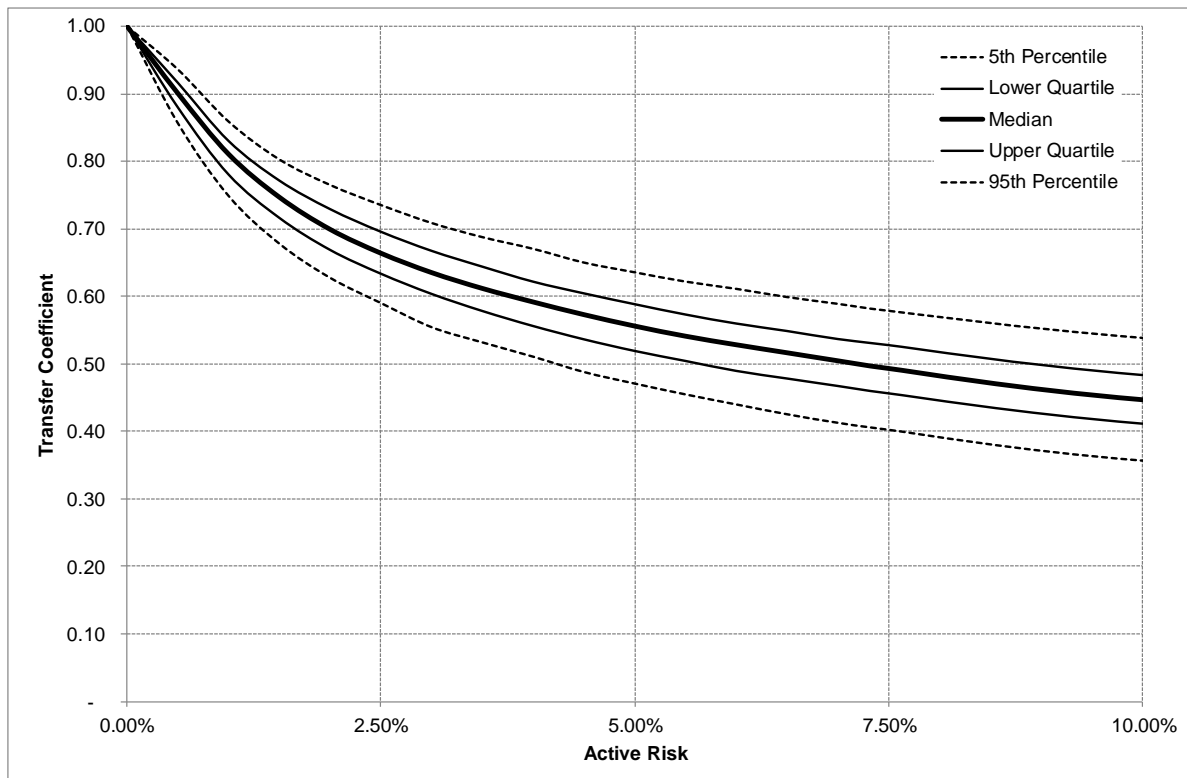
In all figures, the departure for all optimal constrained portfolios from a perfect, unconstrained transfer coefficient of one decreases as active risk increases. These four figures confirm that the long-only constraint is definitively sub-optimal and that the sub-optimality increases at a decreasing rate with increasing active risk. Furthermore, the evidence here shows that the rate of decay of the transfer coefficient is generally more pronounced with lower cross-sectional dispersion in the investment universe: Figure 7.1 and Figure 7.3 show higher (better) distribution of transfer coefficients than Figure 7.2. and Figure 7.4 respectively.



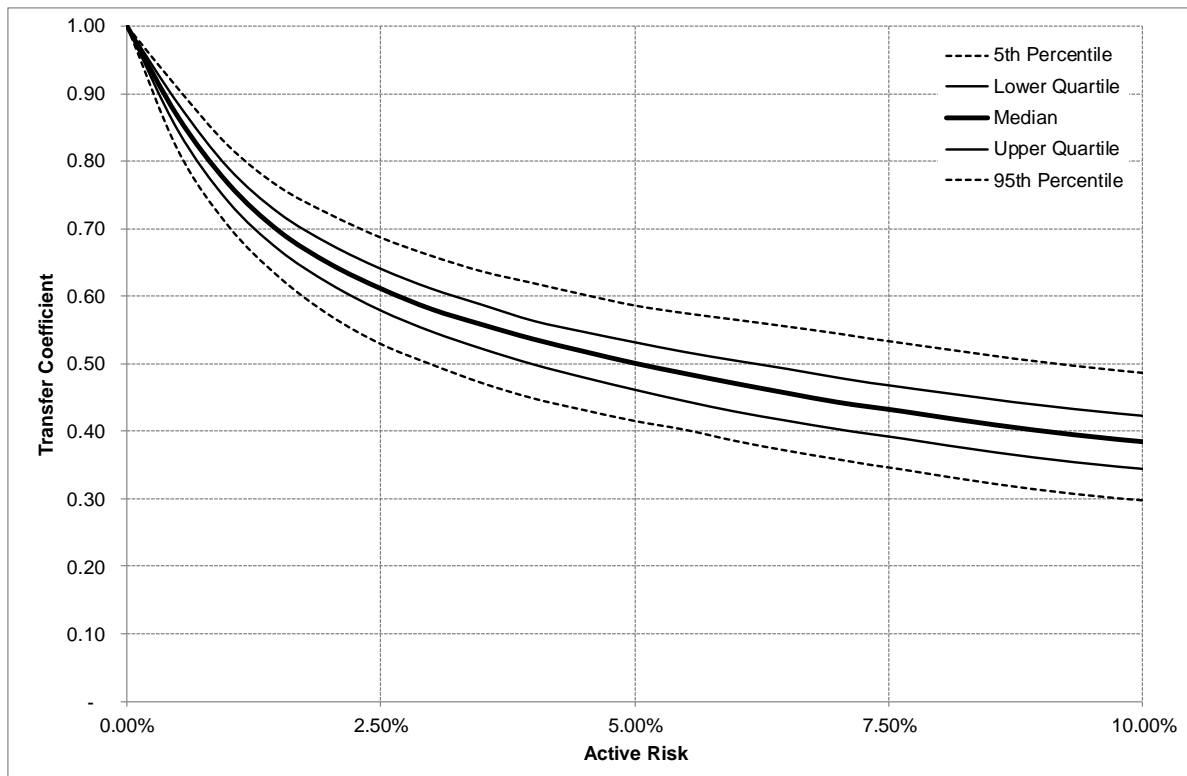
**Figure 7.1: Distribution of TC across all simulated constrained portfolios as a function of active risk-high cross-sectional standard deviation:  $\sigma\sqrt{1-\rho} = 0.50 p.a.$**



**Figure 7.2: Distribution of TC across all simulated constrained portfolios as a function of active risk-low cross-sectional standard deviation:  $\sigma\sqrt{1-\rho} = 0.25 p.a.$**



**Figure 7.3: Distribution of TC across all simulated constrained portfolios as a function of active risk (covariance matrix: July 2008)**



**Figure 7.4: Distribution of TC across all simulated constrained portfolios as a function of active risk (covariance matrix: July 2011)**

Figure 7.3 and Figure 7.4 represent the portfolios constructed with covariance matrices sampled historically from the Barra risk model. As such, they demonstrate the quantum of the loss of efficiency that fund sponsors could have expected under these contrasting market conditions. At a very low active risk budget of 50bp (akin to an enhanced index investment product) 90% of the transfer coefficients were between 0.86 and 0.94 in July 2008 and between 0.82 and 0.91 in July 2011. That is, a long-only ALSI fund manager who is almost passive in their management style, can already expect to lose anywhere between 5% and 20% of the value of their forecast when that forecast is implemented into a long-only portfolio, depending on the cross-sectional market dispersion conditions. The range of the transfer coefficients widens and becomes less favourable very rapidly with increasing active risk budget. At a fairly active risk budget of 4%, 90% of the transfer coefficients were between 0.51 and 0.67 in July 2008 and between 0.45 and 0.62 in July 2011 – a substantial loss of implementation efficiency, particularly in the more recent, low cross-sectional volatility market conditions. This further implies that, if the ex ante investment views are correlated to any extent with subsequent realised views (i.e. if IC is greater than zero), these constrained portfolios will earn anywhere between roughly 45% and 70% of the profits they ought to, depending on the particular investment view and the prevailing cross-sectional market dispersion conditions.

The inefficiency of the constrained portfolios represented by their transfer coefficients is more pronounced at higher active risk levels (as anticipated in Chapter 3) and in lower cross-sectional dispersion conditions (as anticipated in Chapter 4). The evidence presented in Figure 7.3 and Figure 7.4 provide support for the relative efficiency and likely ex post success of skilled active managers (for whom IC is greater than zero) who manage low active risk portfolios such as “enhanced index” products rather than aggressive active

risk mandates. However, the likelihood and size of the transfer coefficient in an ALSI environment, even at very low active risk budgets and a favourable, high cross-sectional variance market is substantial. As demonstrated in Chapter 6, a relaxation of the long-only constraint without necessarily increasing the active risk budget, could substantially improve the efficiency of actively managed portfolios as well as their subsequent profitability.

### **7.3.2 Optimal gearing**

The transfer coefficients presented in the previous section were substantially smaller than one with implications for the ability of skilled, long-only fund managers to efficiently glean profits for their investors. Relaxing or removing the long-only constraint provides a possible solution to the inefficiency imposed by the long-only, highly concentrated investment environment that is the ALSI (as discussed in Chapter 3 and again in Chapter 6). In this section, we remove the long-only constraint entirely to measure the gearing of the optimal unconstrained portfolios, which necessarily have transfer coefficients of one. By comparing the extent to which constrained portfolios fall short of a perfect transfer coefficient (the analysis in Section 7.3.1) with the optimal gearing of the unconstrained portfolios under the same conditions, we can observe the extent of the short extensions that are likely to be required at various active risk targets in order to achieve a maximum transfer coefficient (i.e. transfer coefficient of one).

The optimal gearing is calculated by simply aggregating the size (in absolute terms) of the investment weights of each optimal unconstrained portfolio (refer Equation 7.3). Self-financing, long-only portfolios, such as those formed in this chapter, are required to have

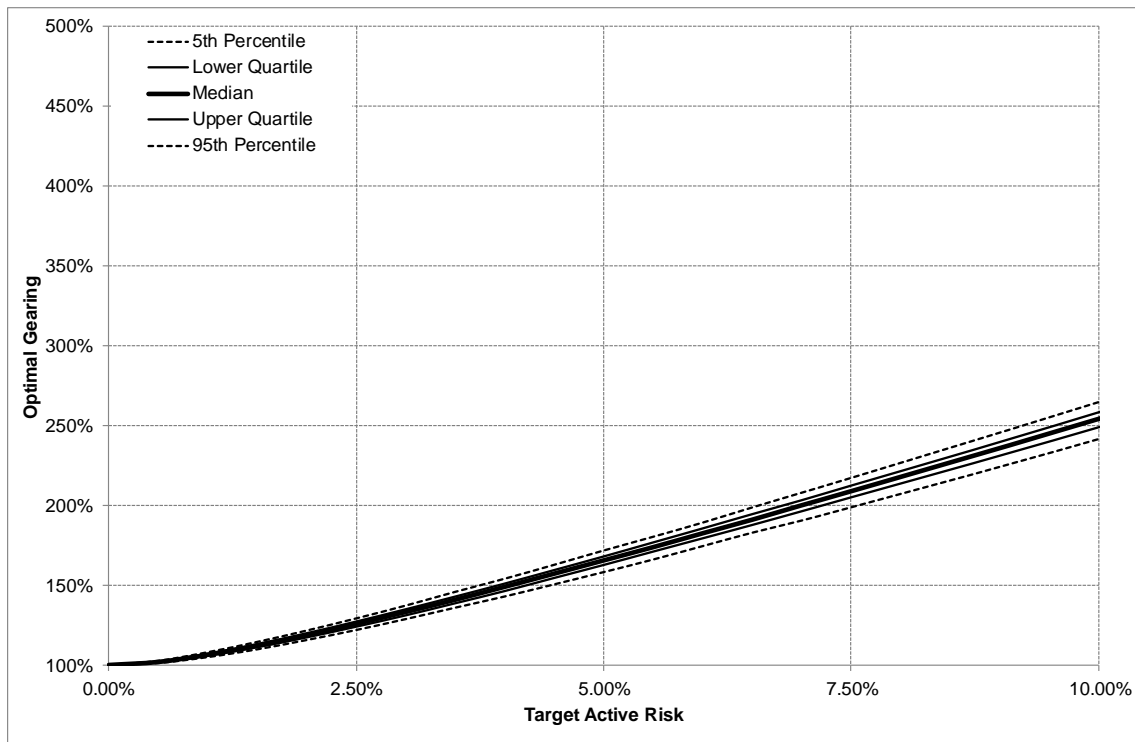
gearing equal to one: their investment weights are not permitted to be smaller than zero or greater than one and the sum of their investment weights must add to 100%.

Unconstrained portfolios as defined in this chapter are able to incorporate short sales (negative weights) and unlimited positive weights in order to achieve their target active risk. Optimal unconstrained portfolios will therefore have gearing greater than or equal to one.

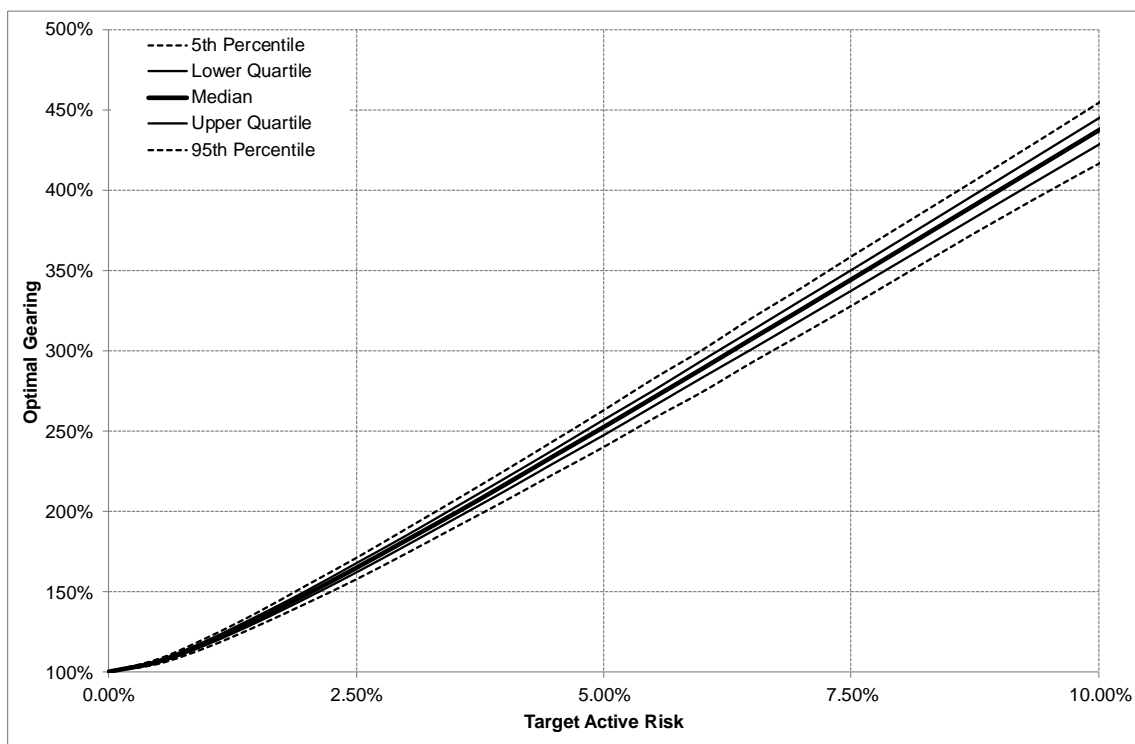
### Equation 7.3: Gearing

$$\sum |w_{ai}|$$

The evidence presented here does not use the same metric as that of Table 6.1 and Table 6.2 of Chapter 6. Those tables provided the aggregate of likely (i.e. mean) short positions across all stocks, conditional on the position being short. In this section, the distribution of the total aggregate gearing of each simulated optimal unconstrained portfolio is calculated and displayed for each of the four covariance matrices.

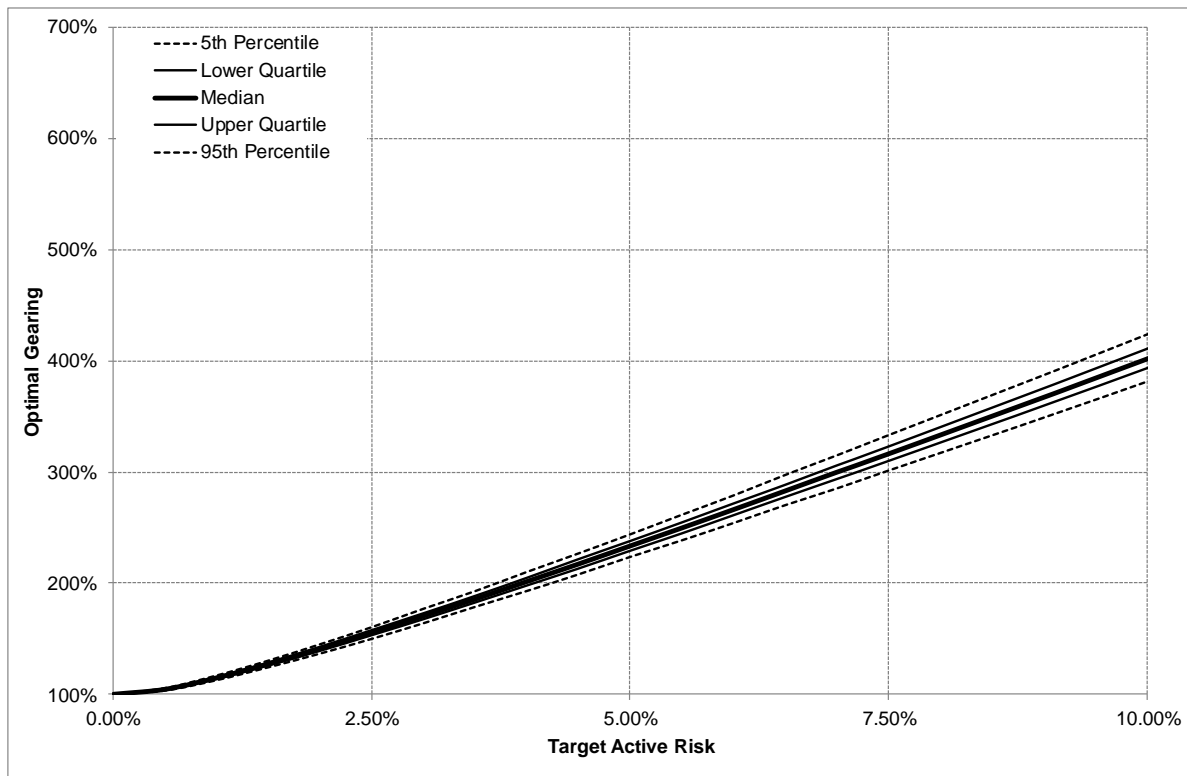


**Figure 7.5: Distribution of optimal gearing across all simulated unconstrained portfolios as a function of active risk - high cross-sectional standard deviation:**  
 $\sigma\sqrt{1-\rho} = 0.50 p.a.$

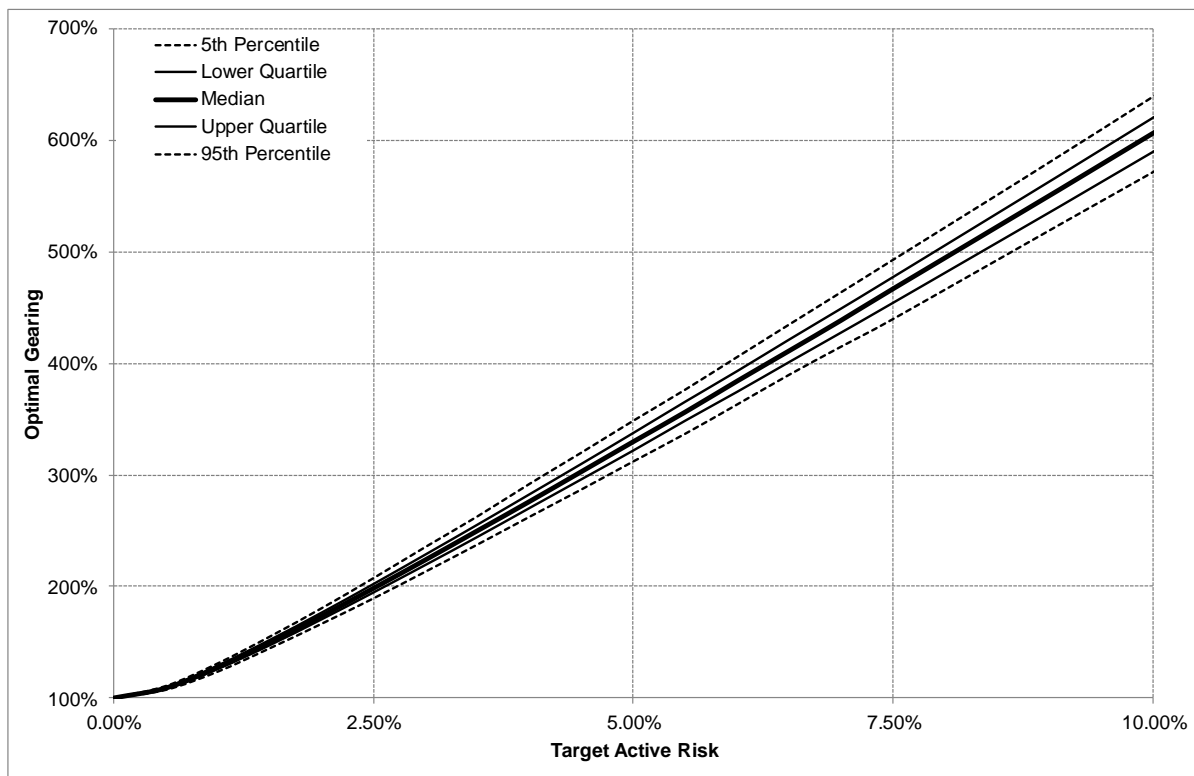


**Figure 7.6: Distribution of optimal gearing across all simulated unconstrained portfolios as a function of active risk- low cross-sectional standard deviation:**  
 $\sigma\sqrt{1-\rho} = 0.25 p.a.$





**Figure 7.7: Distribution of optimal gearing across all simulated unconstrained portfolios as a function of active risk (covariance matrix: July 2008)**



**Figure 7.8: Distribution of optimal gearing across all simulated unconstrained portfolios as a function of active risk (covariance matrix: July 2011)**

As expected, all four figures (Figure 7.5, Figure 7.6, Figure 7.7, and Figure 7.8) demonstrate that greater gearing is required with greater target active risk in order to achieve an optimal unconstrained portfolio with a transfer coefficient of one. Furthermore, as was evidenced in Chapter 6, the amount of gearing is also greater when the cross-sectional variance of the market is lower, all else being equal. In the illustrative examples using simplified covariance matrices (Figure 7.5 and Figure 7.6), a modest short extension of 20% (a 120/20 investment product) would enable optimal construction of an investment portfolio with a 1% active risk budget in most cases (i.e. in Figure 7.5, 95% of the simulated optimal portfolios with 1% active risk required gearing of no more than 120%). An 8% short extension would be sufficient to achieve the same risk budget optimally in a market more favourable for active risk taking (i.e. in Figure 7.6, 95% of the simulated optimal portfolios with 1% active risk required gearing of no more than 108%). With larger active risk targets, such as 5%, the gearing required to achieve an optimal and efficient portfolio is markedly larger and differs substantially with cross-sectional variance conditions.

The historically sampled risk matrices (represented in Figure 7.7 and Figure 7.8) demonstrate the large amount of gearing required to achieve a transfer coefficient of one. For example, a 130/30 investment product allowing a 30% total short extension was only sufficient across all simulations at a 1.5% active risk target in July 2008, and a 1% active risk target in July 2011. To achieve risk targets above 3.5% in July 2008 or about 2% in July 2011 across all simulations with maximum implementation efficiency would have required a maximum gearing allowance of double i.e. 100% of the investment held in short positions and 200% of the investment in long positions. The evidence presented in this section therefore suggests that, although short extensions will undoubtedly improve the

capacity of active managers to achieve normative active risk budgets, especially under low cross-sectional variance conditions, short extensions less than 100% are unlikely to be sufficient to achieve optimal transfer coefficients on a concentrated investment universe such as the ALSI.

### **7.3.3 Active share**

The analyses of simulated portfolios on the ALSI thus far have shown that optimal constrained portfolios are markedly inefficient in their transfer of the investment view to the portfolio, particularly at higher active risk targets. Section 7.3.2 illustrated the large amount of gearing that would be required to achieve optimal portfolios at each level of active risk. In an unconstrained portfolio, gearing enables the enlargement of active positions which are consistent with the investment view ( $TC=1$ ) to achieve any active risk target. When long-only portfolios are constructed, higher active risk targets must be achieved by other means, such as taking active positions in higher active risk-incurring investments that are less than perfectly consistent with the investment view. The resulting constrained optimal portfolio is a combination of active weights of which some represent the manager's investment view and some are a compromise of the investment view in order to achieve the required amount active risk. It is this compromise that reduces the transfer coefficient below one and can result in realised performance that is less than consistent with the manager's forecasting success.

The portfolio's active risk (refer Equation 4.3) is an ex ante estimate of the risk which the active weights incur as a portfolio but it cannot distinguish between large active weights in stocks with low risk and small active weights in stocks with large risks. Cremers and

Petajisto (2009) introduced the “active share” metric, which measures the share of portfolio holdings that differ from the benchmark holdings (Equation 7.4).

**Equation 7.4: Active share**

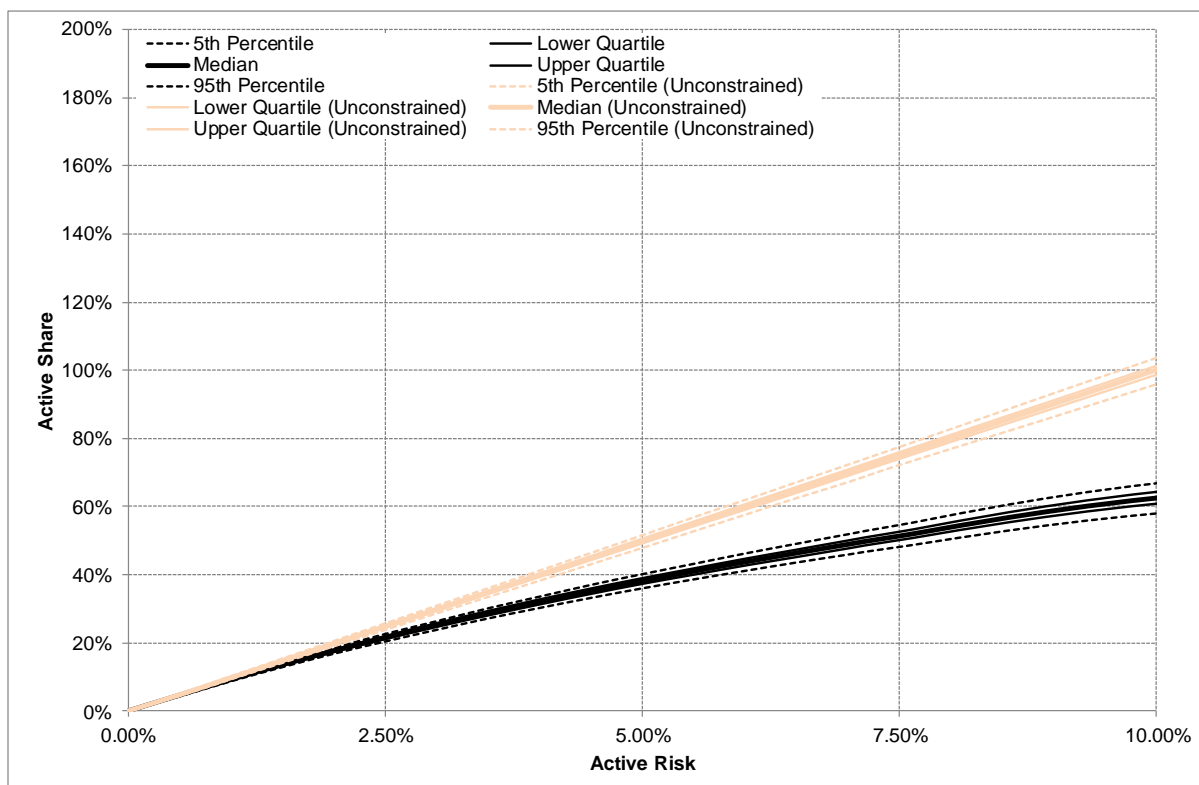
$$\frac{1}{2} \sum |w_a|$$

Active share is, in some ways, easier to interpret than the active risk metric: it does not take into account the complex multivariate nature of an investment portfolio that allows for the diversification of risk but instead active share simply measures the size of the active positions. Since all active weights must add to zero, active share adds the absolute value of the active positions and divides by two to avoid a doubling of scale. In their study of US equity funds from 1980 to 2003, Cremers and Petajisto (2009) found that active share was positively related to subsequent realised performance. They found that funds with high active share tended to outperform their benchmarks and exhibit significant persistence in performance. Cremers and Petajisto (2009) recommended this metric as complementary to the active risk measure as an indicator of the extent of the fund managers’ activity.

In the context of this study, the active share metric enables us to measure the aggregate size of the active positions independent of the scaling effect that the cross-sectional variation of the market has on the active risk measure. Of course, the active weights are dependent on the cross-sectional variance to some extent, as demonstrated in Chapter 6. Nevertheless, the active share metric, unlike the active risk metric, can be measured without the complex interaction of the covariance matrix and provides us with a better

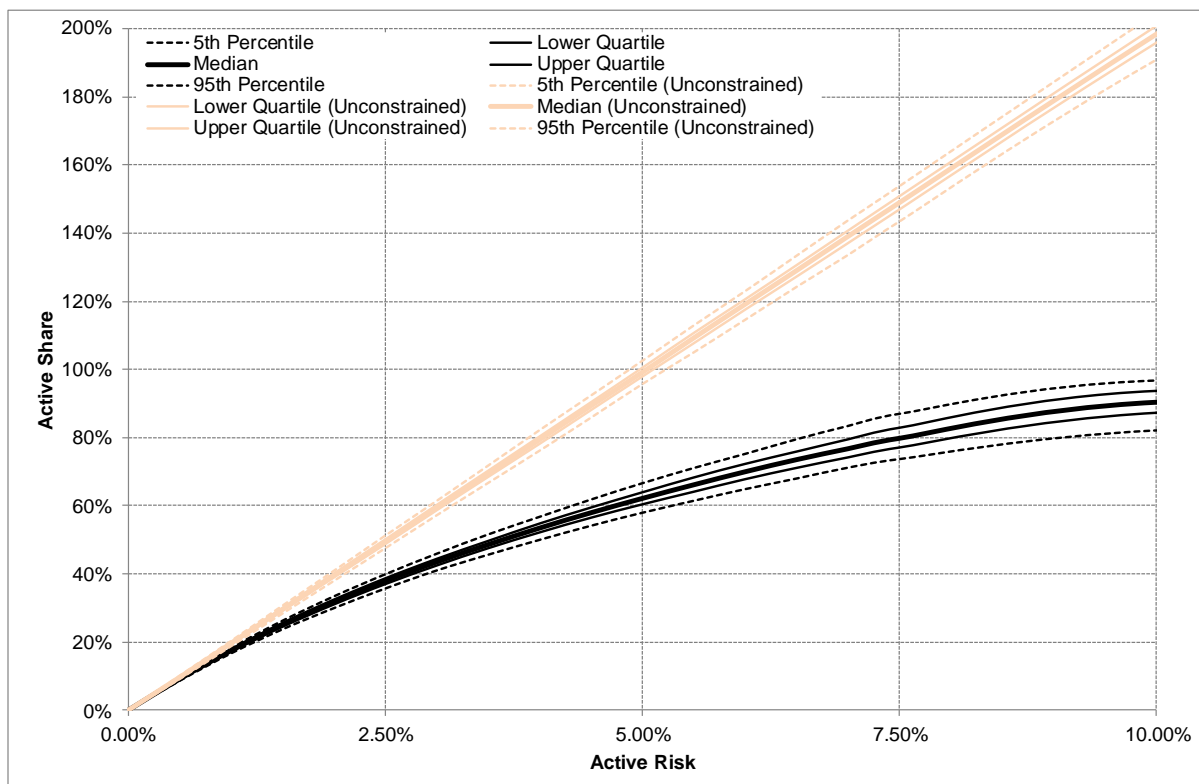
indication of the size of the positions within the fund. This enables a comparison across various active risk targets and market risk conditions on a more reliable scale.

Figure 7.9, Figure 7.10, Figure 7.11 and Figure 7.12 demonstrate the distribution of active share across all simulated alphas and their corresponding optimally constructed portfolios under each of the covariance matrices. The pink lines in each case represent the active share of the optimal unconstrained portfolios. Without the restriction on short sales, each set of alphas is represented by a set of unconstrained optimal active weights that are linearly scaled to achieve the necessary active weights (refer Equation 2.7). The distributions of active share across the unconstrained optimal portfolios that represent all the alpha simulations demonstrate the geared nature of these portfolios: the size and distribution of active share increases with increased active risk budget. In keeping with the findings of Chapter 4, the rate of increase in active share required by higher target active risk is higher when the cross-sectional dispersion in the investment universe is low (Figure 7.10 and Figure 7.12) than when the cross-sectional dispersion high (Figure 7.9 and Figure 7.11). It is important to note that this increase is not a consequence of a change in view, but rather the increased active risk budget.

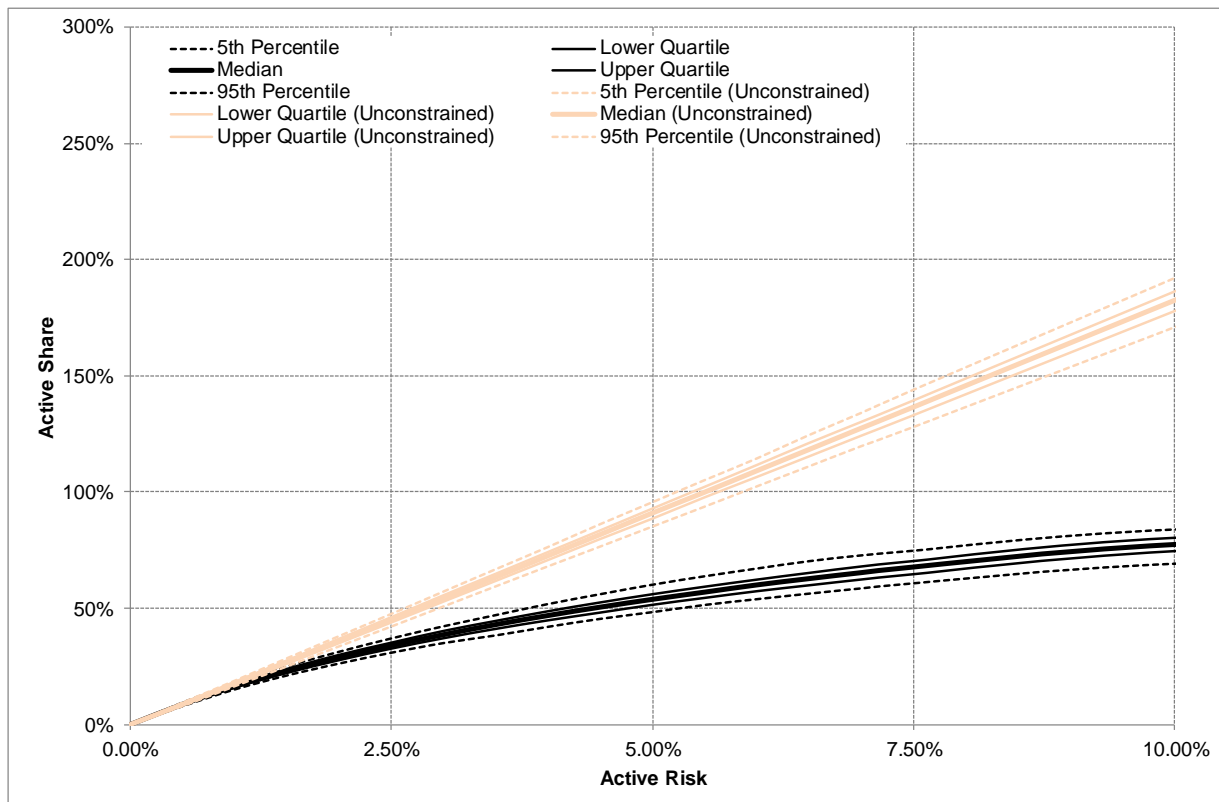


**Figure 7.9: Distribution of active share across all simulated constrained portfolios as a function of active risk - high cross-sectional standard deviation:**

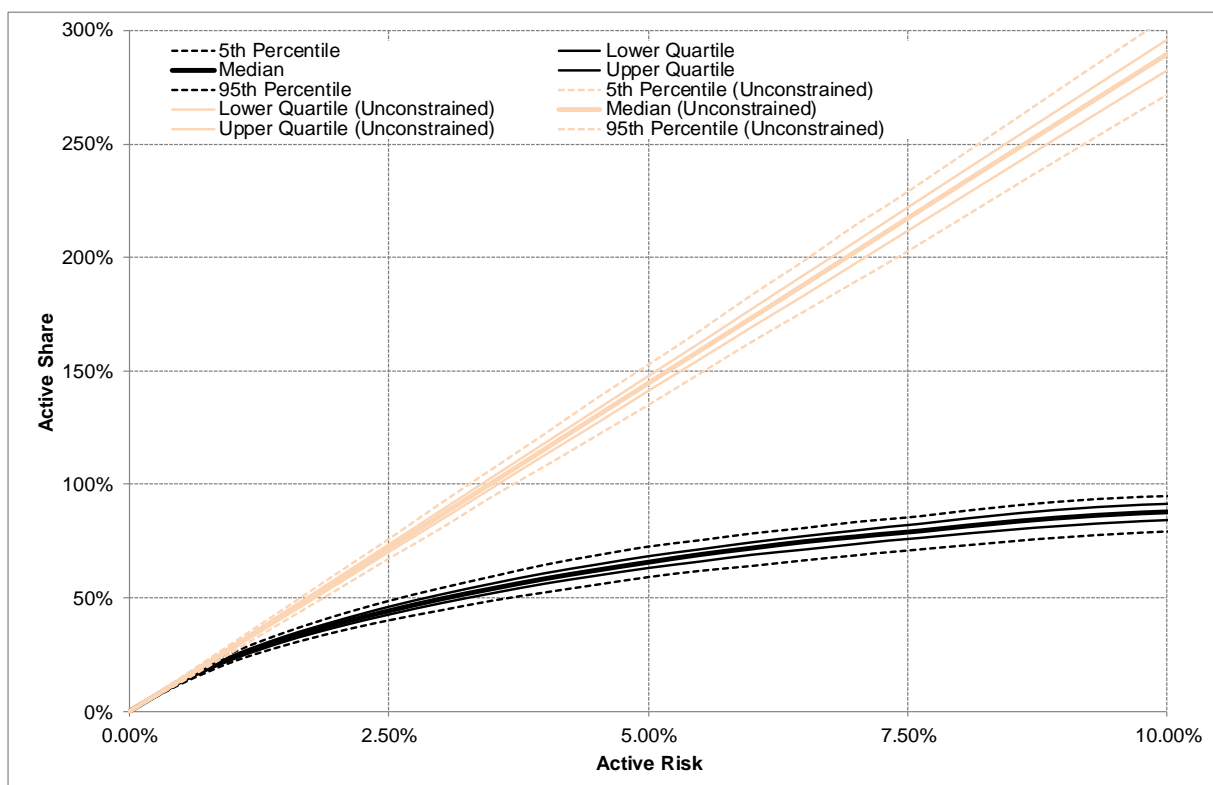
$$\sigma\sqrt{1-\rho} = 0.50p.a.$$



**Figure 7.10: Distribution of active share across all simulated constrained portfolios as a function of active risk- low cross-sectional standard deviation:  $\sigma\sqrt{1-\rho} = 0.25p.a.$**



**Figure 7.11: Distribution of active share across all simulated constrained portfolios as a function of active risk (covariance matrix: July 2008)**



**Figure 7.12: Distribution of active share across all simulated constrained portfolios as a function of active risk (covariance matrix: July 2011)**

The black lines in each of Figure 7.9, Figure 7.10, Figure 7.11 and Figure 7.12 represent the active share of the optimal constrained portfolios on the same scale as their unconstrained counterparts. In each figure, portfolios on the same vertical line have the same active risk but different active share. Although the distribution of active share across constrained portfolios increases with increasing active risk, it increases at a decreasing rate. In each case, the distribution of active share for unconstrained portfolios is lower than that of the unconstrained portfolios and the departure of the two distributions increases with increasing active risk. Furthermore, the separation of these two distributions is more pronounced when the cross-sectional variance was low (Figure 7.10 and Figure 7.12) than when it was high (Figure 7.9 and Figure 7.11). The long-only manager can only extend their active positions as far as the long-only constraint allows. Thereafter, the active risk of the portfolio can only be increased by financing increasing positive active positions with a greater number of negative positions, departing from the investment view in favour of positions that serve to increase the active risk of the portfolio. The active share of constrained portfolios demonstrates the decreasing rate of “activity” on the part of the optimal constrained fund manager: as active risk increases and the long-only constraint becomes increasingly binding, so the size of the active weights in the portfolio become more diffused,

#### **7.3.4 Concentration**

Strongin *et al.* (2000) popularised the use of the effective number of stocks as an indication of benchmark concentration. In this study, the same metric is applied to the composition of all the optimal portfolios. Note that the definition in Equation 7.5 is consistent with the definition in Equation 2.13: the definition is repeated here in a way that

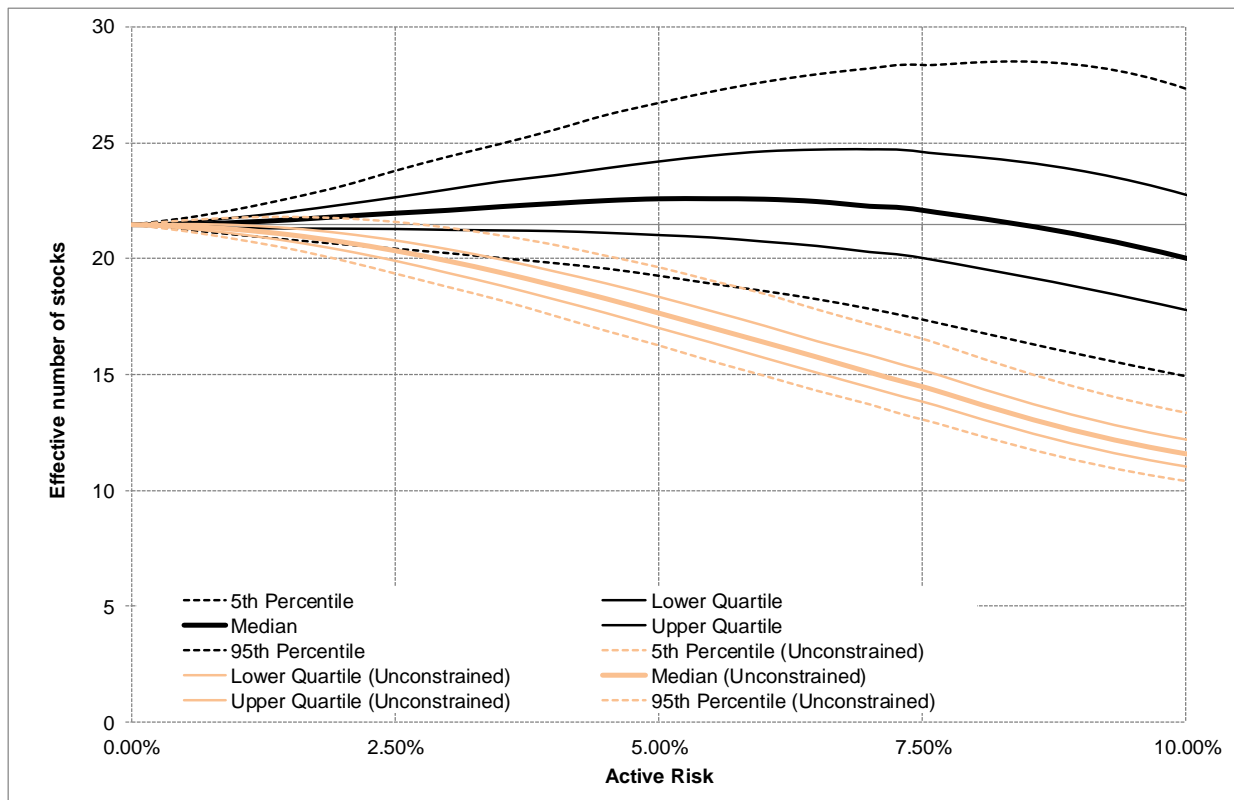


is consistent with the terminology of this chapter but, as in Equation 2.13, concentration is measured using the actual investment weights as opposed to the active weights.

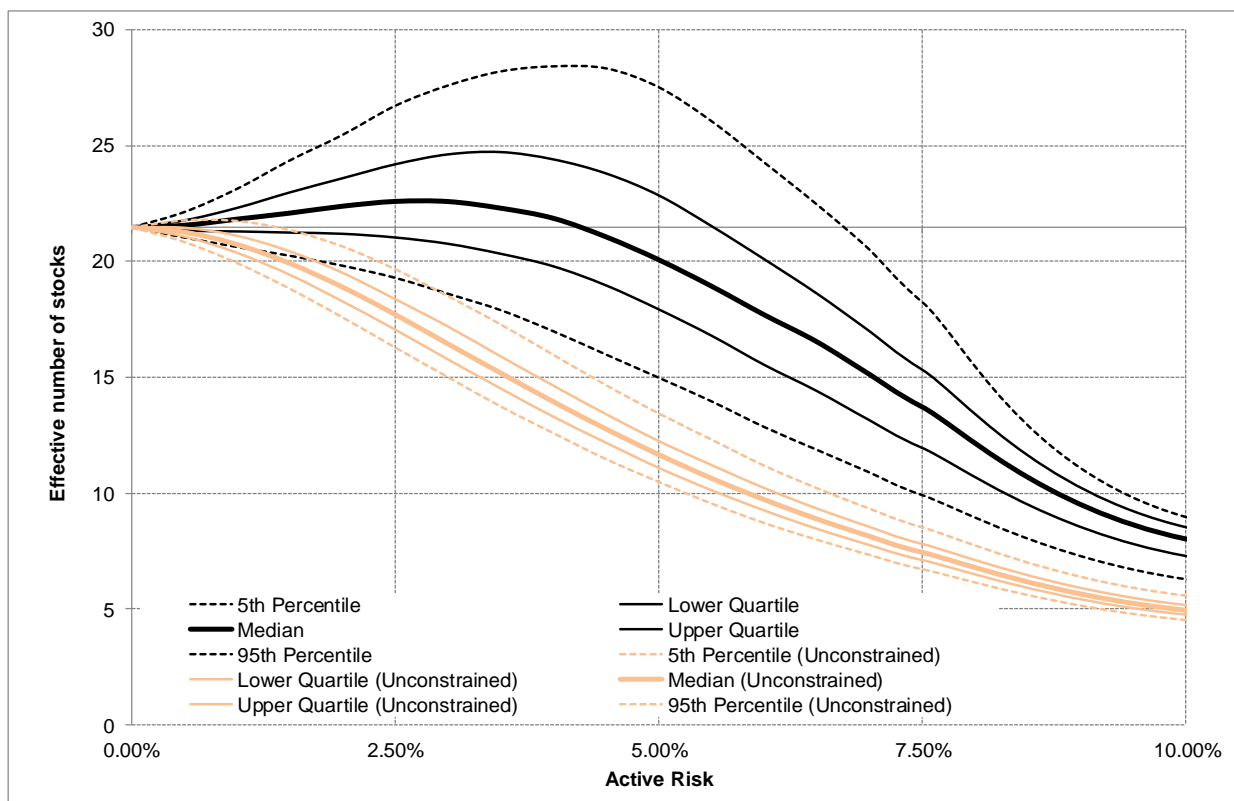
#### Equation 7.5: Effective number of stocks

$$\frac{1}{\sum (w_{a,i} + w_{b,i})^2}$$

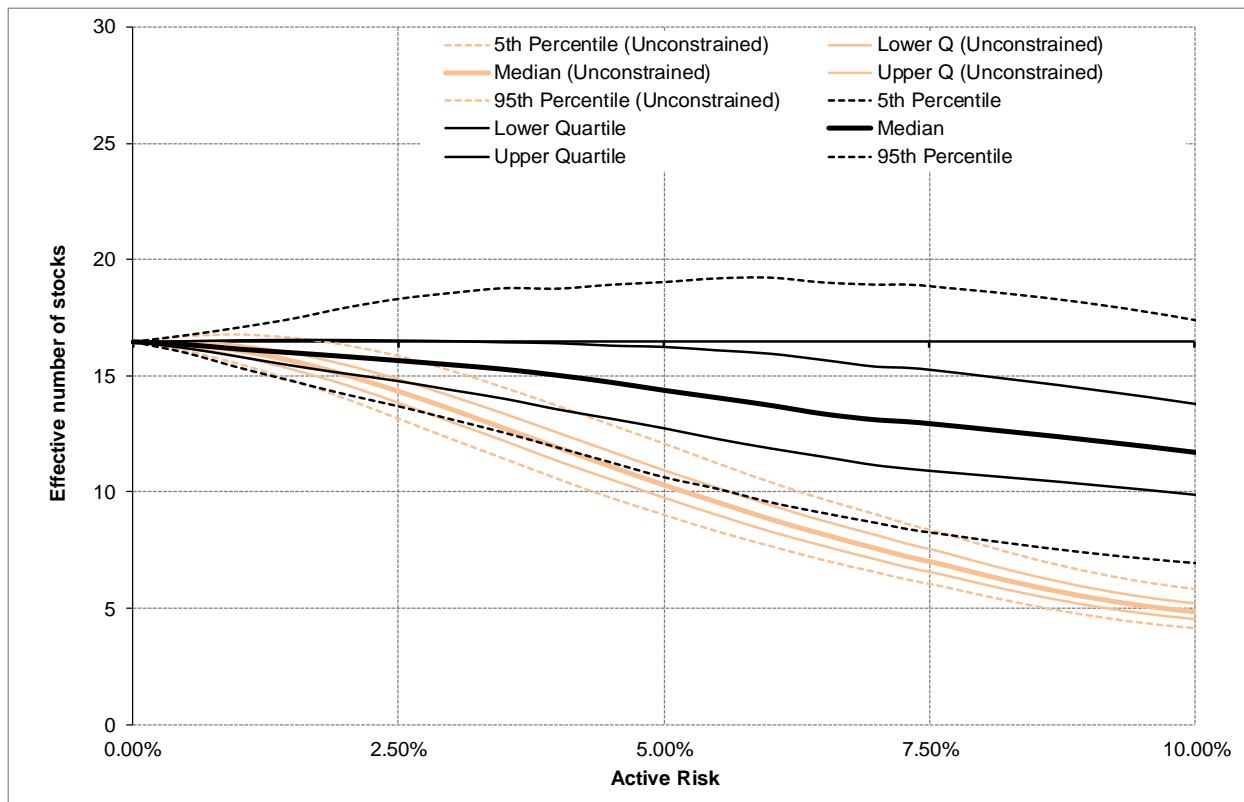
The effective numbers of stocks of the optimal portfolios are compared with and without the long-only constraint, using all four covariance matrices. Without restrictions on short sales, the optimal solution for increasing the active weight of a portfolio is to increase the size of the active weights of the portfolio, which, in turn, amounts to an increased concentration of the portfolio. The pink lines in Figure 7.13 describe the distribution of the concentration of the optimal unconstrained portfolios as a function of increasing active risk. These lines begin at the concentration of the benchmark portfolio (zero active risk) which in July 2011 was 21.5 effective stocks. The concentrations of the optimal portfolios vary with the simulated alphas. As a whole, there is a marginal increase in concentration under these high cross-sectional dispersion conditions. By contrast, when cross-sectional dispersion is low, the carrying capacity for active risk is diminished and Figure 7.14 shows a sharper increase in concentration among optimal unconstrained portfolios. Using these simplified, two-parameter covariance matrices and the same benchmark, the effect on the concentration of optimal portfolios is clear: greater active risk budgets require greater investment concentration, even when gearing by way of short sales is permissible. The smaller the market's carrying capacity for active risk (as measured by the cross-sectional standard deviation of its assets), the greater the increase in concentration is required to satisfy the active risk budget.



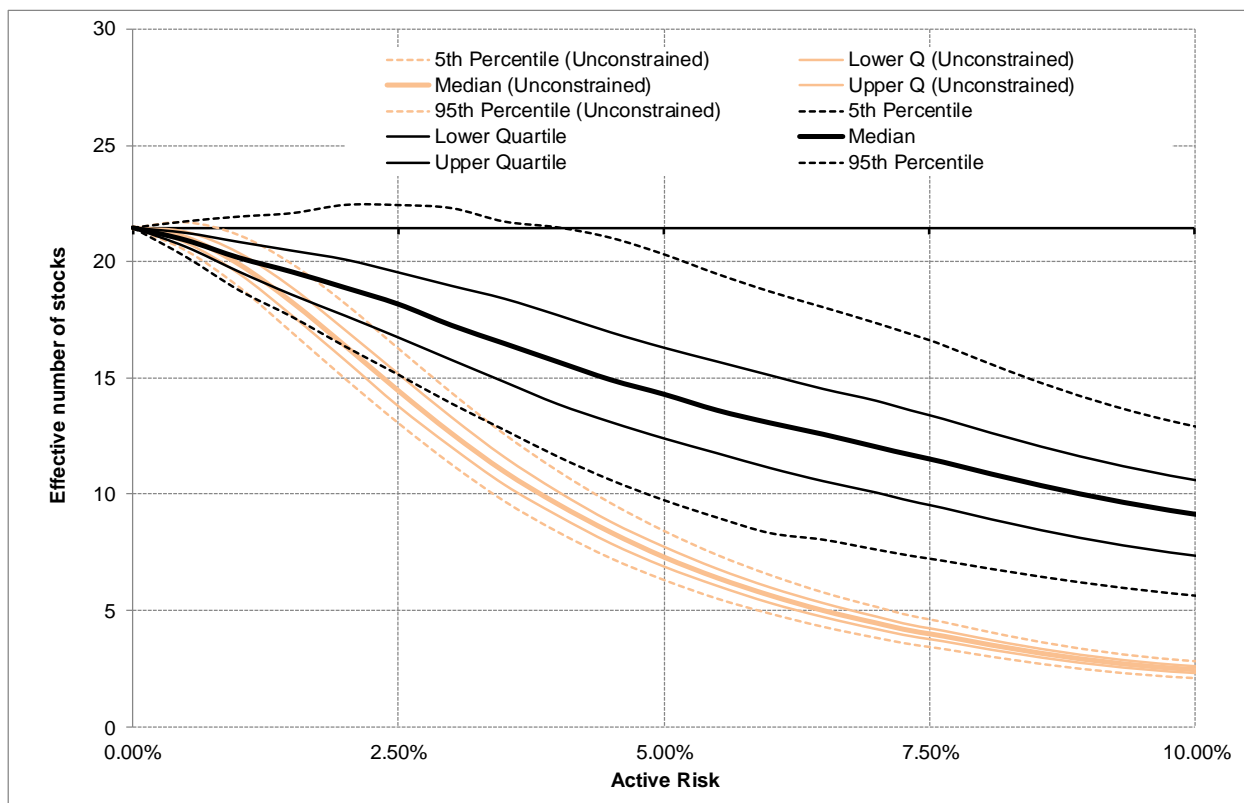
**Figure 7.13: Distribution of concentrations of all simulated constrained portfolios as a function of active risk (high cross-sectional standard deviation:  $\sigma\sqrt{1-\rho} = 0.50 p.a.$ )**



**Figure 7.14: Distribution of concentrations of all simulated constrained portfolios as a function of active risk - low cross-sectional standard deviation:  $\sigma\sqrt{1-\rho} = 0.25 p.a.$**



**Figure 7.15: Distribution of concentrations of all simulated constrained portfolios as a function of active risk (covariance matrix: July 2008)**



**Figure 7.16: Distribution of concentrations of all simulated constrained portfolios as a function of active risk (covariance matrix: July 2011)**

The black lines in Figure 7.13 and Figure 7.14 contrast with the optimal unconstrained solutions and demonstrate the effect of long-only constraints in each of these environments. Once again, the starting point is the benchmark portfolio with zero active risk and an effective number of stocks of 21.5. With an increase in active risk, the restriction against short sales hinders the ability to simply increase the active bet. As the maximum allowable negative active weight is reached for each of the negatively held assets ( $w_{a,i} = -w_{b,i}$ ), different assets must be used to amass additional active risk. At lower active risk targets, these additional negative positions, which are inconsistent with the fund manager's view of the investment universe, create more diversified portfolios. Interestingly, at greater active risk levels, the capacity of these inconsistent negative positions to finance increasing allowable active positions (usually positive positions) starts to diminish, at which point the concentration of the optimal constrained portfolios increases again. Contrasting the distribution of the optimal constrained portfolios under high (Figure 7.13) and low (Figure 7.14) market dispersion conditions, it is clear that this point is reached at lower active risk budgets when the market's capacity for active risk is low. In a long-only portfolio at higher active risk and/or in a market offering low cross-sectional variation, many of the smaller stocks are sold to finance larger active bets. The many zero investment weights that result create a highly concentrated portfolio.

### 7.3.5 Weight not taken

Clarke *et al.* (2006) defined the “weight not taken” as a measurement of the impact of portfolio constraints (refer Equation 7.6). The weight not taken in any asset is the difference between the optimal constrained weight and the expected weight given the overall impact of the constraints (represented by the transfer coefficient). The transfer coefficient therefore represents the impact of the constraints on the whole portfolio of

assets whereas the weight not taken measures the asset-specific impact of these constraints.

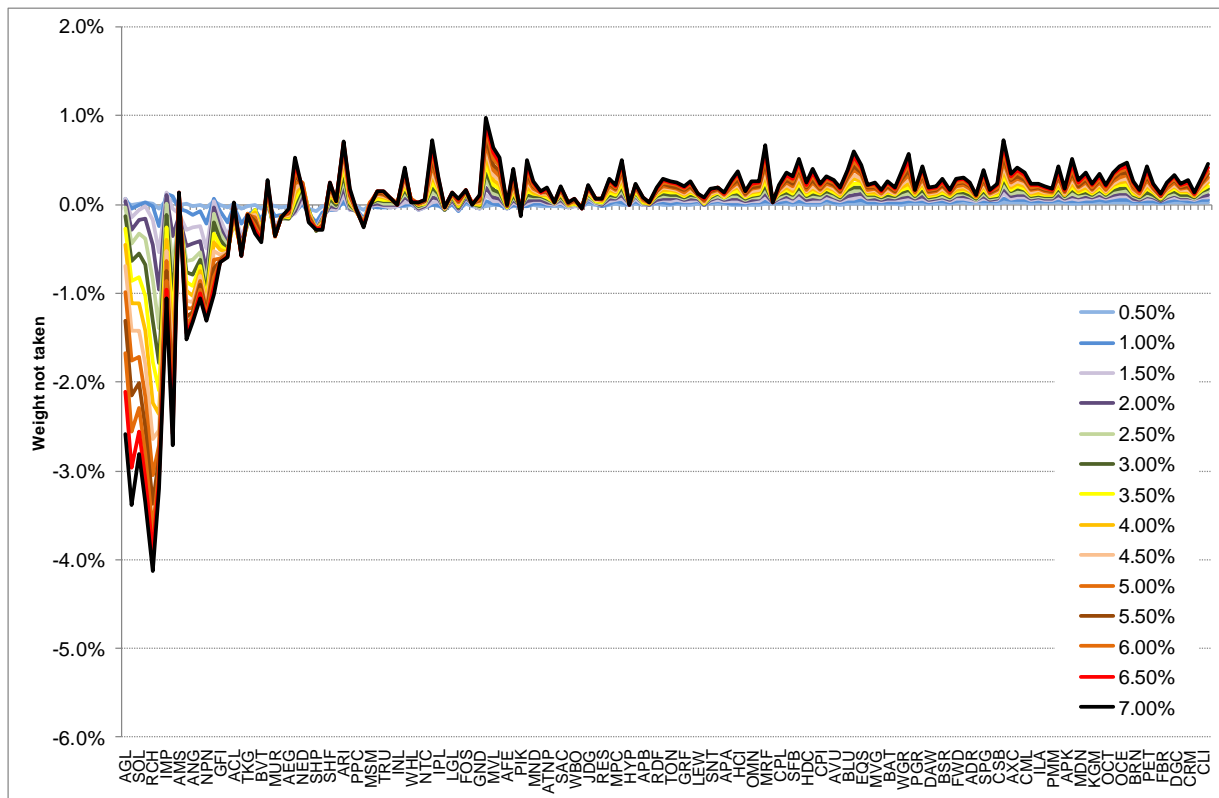
### Equation 7.6: Weight not taken

$$c = w^c - TC \cdot w^{unconstrained}$$

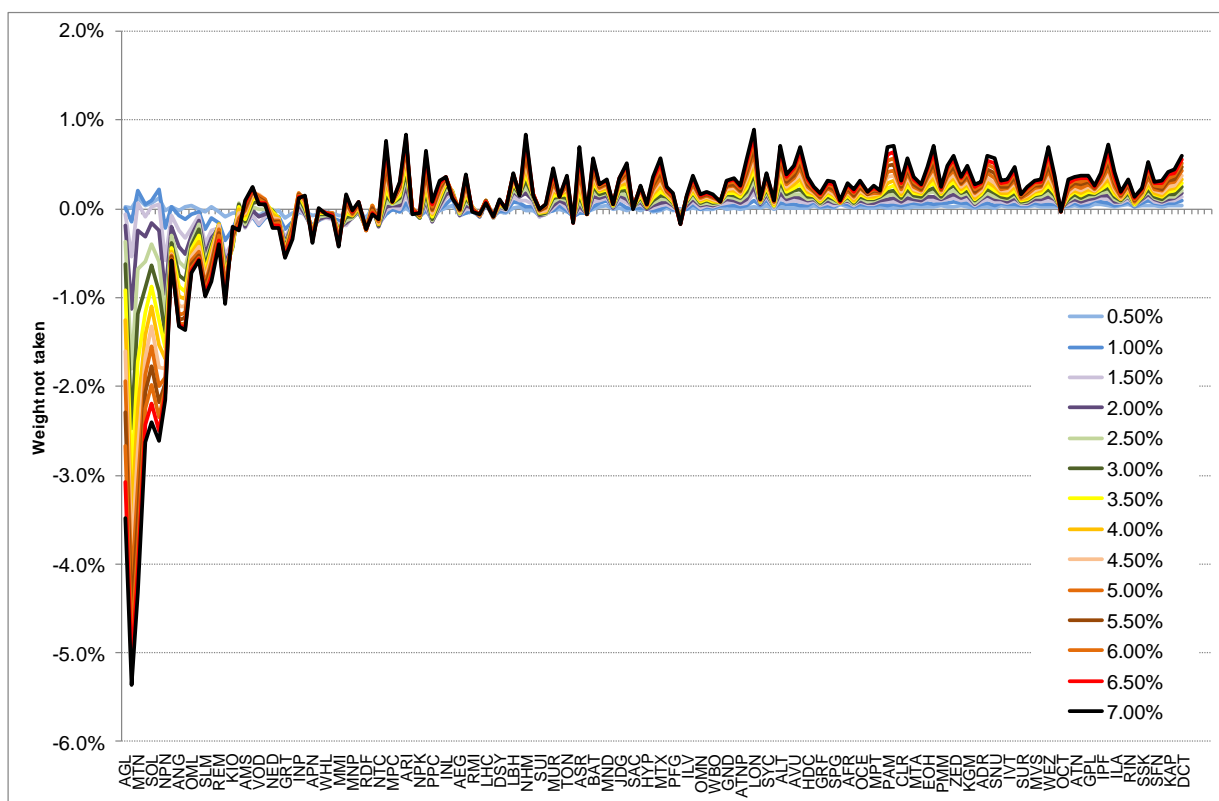
where  $c$  is the weight not taken across all assets and  $w^c$  and  $w^{unconstrained}$  are the constrained and unconstrained optimal portfolio weights respectively.

Clarke *et al.* (2006) derives a realised noise coefficient to measure the noise created by constraints in the realised performance of the portfolio. This noise coefficient is a function of the weight not taken ( $c$ ), the covariance matrix and the realised returns. For this reason, the weight not taken is a fitting way to measure the asset level impact of constraints across various risk conditions and is applicable even ex post. The aim in this section is to observe the effective and unintended size bias implied by the long-only constraint by observing the relationship of the weight not taken and the relative size of the underlying assets.

Figure 7.17 and Figure 7.18 comprise the average, across all simulated investment views, of the weight not taken per stock using a high and low cross-sectional volatility covariance matrix respectively. The 160 stocks of the ALSI are listed from left to right in each figure, arranged from largest to smallest in terms of their weight in the ALSI at the time. Each line represents a particular active risk target.



**Figure 7.17: Average weights not taken of all simulated constrained portfolios (covariance matrix: July 2008)**



**Figure 7.18: Average weights not taken of all simulated constrained portfolios (covariance matrix: July 2011)**

The evidence presented in these figures suggests that, on average and after taking into account the inefficiency of the long-only constraint, long-only managers are less positively invested in large stocks than they might otherwise be and more negatively invested in smaller stocks. In order for long-only managers to take a strong positive active position, a balancing negative position must be found. As shown in Chapter 3, there is a far greater range of positive positions than negative positions in a concentrated market when short sales are not permitted. Consequently, a large collection of small negative active positions is required to finance each substantial positive position. It is therefore more likely that, on average, optimal long-only managers will take smaller positions in large stocks than their forecasts would suggest and larger positions in small stocks. This bias away from positive investment in large stocks is more pronounced when higher active risk targets are set and when a lower cross-sectional dispersion environment exists.

All the other metrics used in this section have been portfolio measures describing the effect that the long-only constraint has on the portfolio as a whole. The weight not taken enables a demonstration of the general bias against greater investment in larger securities and smaller investment in bigger securities that is created by the long-only constraint.

## **7.4 CONCLUSIONS**

Investment portfolios must be constructed in such a way that they satisfy the objectives of the client, the restrictions of the investment mandate and the investment views of the fund manager. Some of the friction and compromise inherent in investment portfolios striving to meet these requirements, has been documented in this chapter making use of simulated investment views and optimised investment portfolios. The contribution of this

chapter is to document the size and nature of various inefficiencies that are implied by the long only constraint within the context of a concentrated investment universe as represented by the ALSI and the varying cross-sectional dispersion of the ALSI's return offering.

In particular, the analysis in this chapter shows the material sub-optimality of long-only portfolios as measured by their transfer coefficients and the unreasonably large amount of gearing that would be required to equate these portfolios with their unconstrained counterparts at higher active risk levels and under conditions of low cross-sectional dispersion. In order to achieve increasing active risk targets, the analysis shows that long-only portfolios are compelled to be less active than their unconstrained counterparts are and to become more "active" at a decreasing rate as measured by their active share.



## Chapter 8 : Conclusions and Recommendations

The research presented in this thesis speaks to the management of fund managers, specifically the setting of investment objectives and constraints, and the subsequent assessment of investment performance. In particular, the influence of the following features of South African pension fund management have been explored:

- i) the concentration of the South African equity offering;
- ii) the restriction against short investment positions; and
- iii) the varying cross-sectional dispersion of realised returns.

The success (and failure) of fund managers' activities is obscured by the inherent randomness and volatility of the market in which they exercise their forecasting skill.

When the investment constraints, the risk budget and the investment universe required by the investment mandate are ill-suited to each other, these mandates can materially change the way in which fund managers are able to act on their investment forecasts and result in suboptimal portfolio construction and a distortion of subsequent portfolio performance amidst this randomness.

The **cross-sectional volatility** of asset returns in an investment universe represents a carrying capacity for active risk taking: the more dispersed the returns of investments are, the more opportunities there are for active managers to perform differently from their competitors and the benchmark. Chapter 4 shows that cross-sectional volatility on the ALSI has changed substantially over time as have the opportunities for superior active performance. Chapter 5 shows that the dispersion in realised fund returns is closely related to the dispersion of the underlying securities in which they invest, implying

heteroscedasticity in the time series of fund performances as well. The ex post performance of competing portfolio managers should therefore be corrected for the varying cross-sectional risk of their investment environment in order to more accurately and fairly assess the extent of their success and skill. This is particularly true when using metrics related to regression or t statistics such as the Sharpe or Information Ratios as all these methods assume homoscedasticity in the series to which they are applied.

Chapter 5 demonstrates the success of a simple weighted adjustment to performance time series using benchmark dispersion in correcting for heteroscedasticity. This weighted adjustment allows manager performances to be compared on a more level playing field by considering the heteroscedasticity inherent in their investment environment during the history of the portfolio's performance: returns delivered in periods of high dispersion in the benchmark assets should be weighted less than returns earned in periods of low benchmark dispersion allowing for more efficient and appropriate detection of manager skill. Because the adjustment is a function of the contemporaneous benchmark security dispersion, it is simultaneously measurable, objective and free of sampling differences that might arise from estimating the source of heteroscedasticity directly from the sample of fund performances.

When it comes to the ex ante management of fund managers, when mandate agreements are constructed, cross-sectional volatility should be considered hand-in-hand with risk limits and active risk targets. The higher the cross-sectional volatility, the greater the opportunity for active risk taking, all other things being equal. Conversely, to remain efficient, active risk taking should be reduced during periods of low cross-sectional dispersion. Furthermore, when fund sponsors monitor changes in an ex ante active risk

they should bear in mind that changes in the active risk forecast of a portfolio could be a reaction to changes in cross-sectional dispersion and not the result of changes in the active positions of the fund. Sponsors and their managers should therefore exercise caution when reacting to changes in active risk estimates, mindful of the relationship between cross-sectional dispersion and active risk.

That being said, active fund managers can only fully express their views in an environment where their conviction and level of risk taking are accommodated by their constraints. The restriction against short sales is materially binding. Chapter 3 showed that the long-only constraint, when applied to a benchmark with a large number of small members, implies a substantial asymmetry in the opportunity set of active weights, particularly when the allowable active weights are larger. The higher the allowable active bet sizes and the higher the active risk budget, the less competitive a long-only fund manager can be alongside hedge funds and similarly limited long-short managers. A concentrated benchmark/investment environment such as the ALSI where only a few of the stocks comprise most of the total investment weight exacerbates this competitive disadvantage.

In a long-only, concentrated environment, low risk active strategies provide investors with the best “bang for their buck” because, under these conditions, long-only fund managers have the opportunity to act more fully on their active views across the full cross-section of available securities. By contrast, to require long-only managers to take more aggressive active positions in a concentrated investment environment is, ironically, only to constrain them further in their abilities to express their best active view.

Chapter 6 illustrates the importance of the choice of benchmark and risk budget in determining the distribution of optimal investment weights in the portfolio. The more concentrated the benchmark and the higher the active risk target, the wider the distribution of asset weights and the more binding the typical restrictions on asset weightings will be. Furthermore, constraints on short positions are more binding on assets with low-weightings in the benchmarks illustrating the asymmetric and sub-optimal effect of constraints applied equally to all assets in the investment universe. Chapter 6 confirms the increased gearing allowance required for funds with decreasing cross-sectional dispersion in their investment universe, increasing risk budgets and highly concentrated benchmarks. By implication, the detrimental effect of the long-only constraint on the active fund manager's ability to implement their investment views worsens under the same conditions.

The derivations and simulation techniques used in Chapter 6 describe a way in which mandate authors could calculate the distribution of optimal investment weights to assist them in finding reasonable asset-level constraints that are consistent with their choice of benchmark and risk budget. By implication, mandate authors can use these techniques to determine the extent of the sub-optimality of security-specific restrictions, such as the long-only constraint. Similarly, mandate authors who are considering relaxing the long-only constraint can determine the likely short extension required under various market conditions and risk budgets.

Ultimately, investment portfolios must be constructed in such a way that they satisfy the objectives of the client, the restrictions of the investment mandate and the investment views of the fund manager. Chapter 7 makes use of multiple simulated investment views

and their corresponding optimal portfolio solutions to document the size and nature of various inefficiencies that are implied by the long only constraint within the context of a concentrated investment universe as represented by the ALSI and the varying cross-sectional dispersion of the ALSI's return offering.

Chapter 7 shows the material sub-optimality of long-only portfolios as measured by their transfer coefficients and the unreasonably large amount of gearing that would be required to equate these portfolios with their unconstrained counterparts, particularly at higher active risk levels and under conditions of low cross-sectional dispersion.

By measuring the nature and size of the impact of market concentration, cross sectional return dispersion and restrictions on short sales on the quality of the investment portfolio, the research presented in this thesis provides analysis and techniques which inform and can improve the quality of the relationship between fund manager and fund sponsor. The more appropriate the investment mandate and the monitoring of the fund manager's performance subject to this mandate, the more effective the fund manager's risk-taking on behalf of their investors will be.

## References

- Ang, A., Hodrick J.R., Xing Y. & Zhang X. 2006. The Cross-Section of Volatility and Expected Returns, *The Journal of Finance*, 61 (1), 259 to 299.
- Ankrim, E.M. & Ding, Z. 2002. Cross-Sectional Volatility and Return Dispersion, *Financial Analysts Journal*. September/October, 67-73.
- Bernstein, P.L. 1998. Where, Oh Where Are The .400 Hitters of Yesteryear? *Financial Analysts Journal*. **56**(6), 6-14.
- Bernstein, P.L. 1996. *Against the gods: The remarkable story of risk*. New York: Wiley & Sons.
- Best, M.J., & Grauer, R.R. 1991. On the Sensitivity of Mean- Variance-Efficient Portfolios to Changes in Asset Means : Some Analytical and Computational Results. *The Society for Financial Studies*, **4**(2), 315-342.
- Breusch, T.S. & Pagan, A.R. 1979. A Simple Test for Heteroscedasticity and Random Coefficient Variation. *Econometrica*, **47**(5), September, 1287-1294.
- Broadie, M. 1993. Computing efficient frontiers using estimated parameters. *Annals of Operations Research*, **45**(1), 21-58.
- Buckle, D. J. 2004. How to Calculate Breadth: An Evolution of the Fundamental Law of Active Portfolio Management. *Journal of Asset Management*, **4**, 54-63.
- Buckle, D. J. 2005. Ex Post Reality Versus Ex Ante Theory of the Fundamental Law of Active Management. *Journal of Asset Management*, **6**(1), 21-32.
- Cartillier, J. 2006. African stock markets seek to work in concert, *Mail and Guardian*, 19 September.
- Clarke, R.G., De Silva, H. & Sapra, S.G. 2004. Toward More Information-Efficient Portfolios. *Journal of Portfolio Management*, **31**(1), Fall, 54-63.

- Clarke, R.G, De Silva, H., Sapra, S. & Thorley, S. 2008. Long-Short Extensions: How Much is Enough? *Financial Analysts Journal*, **64**(1), January/February, 16-30.
- Clarke, R.G., De Silva, H. & Thorley, S. 2002. Portfolio Constraints and the Fundamental Law of Active Management, *Financial Analysts Journal*, **58**(5), September/October, 48-66.
- Clarke, R.G., De Silva, H. & Thorley, S. 2005. Performance Attribution and the Fundamental Law. *Financial Analysts Journal*, **61**(5), September/October, 70-83.
- Clarke, R.G., De Silva, H. & Thorley, S. 2006. The Fundamental Law of Active Portfolio Management with Full Covariance Matrix. *Journal of Investment Management*, **4**(3), 54-72.
- Clarke, R.G., De Silva, H. & Thorley, S. 2010. Know Your VMS Exposure. *The Journal of Portfolio Management*, **36**(2), Winter, 52-59.
- Cremers, M. & Petajisto, A. 2009. How Active Is Your Fund Manager? A New Measure That Predicts Performance. *Review of Financial Studies*, **22**(9), 3329-3365.
- De Silva, H., Sapra, S.G., & Thorley, S. 2001. Return Dispersion and Active Management. *Financial Analysts Journal*, **57**(5), September/October, 29-42.
- Goetzmann, W.N., Grinold, R.C., & Kahn, R.N. 1996. Active Portfolio Management. *Journal of Finance*, **51**(4), 1559-1562.
- Goodwin, T.H. 1998. The Information Ratio. *Financial Analysts Journal*, **54**(4), 34-43.
- Gorman, L.R., Sapra, S.G. & Weigand, R.A. 2010. The Cross-Sectional Dispersion of Stock Returns , Alpha, and the Information Ratio. *The Journal of Investing*, (Fall), 113-127.
- Grinold, R.C. 1989. The Fundamental Law of Active Management. *Journal of Portfolio Management*, Spring, 30-37.
- Grinold, R.C. 1994. Alpha is Volatility Times IC Times Score. Real alphas don't get eaten. *Journal of Portfolio Management*, **20**(4), Summer, 9-16.
- Grinold, R.C. 2005. Implementation Efficiency. *Financial Analysts Journal*, **61**, 52-64.

Grinold, R.C. & Kahn, R.N. 2000. *Active Portfolio Management* (2nd ed.). McGraw Hill.

Grinold, R.C., & Taylor, M.P. 2009. The Opportunity Set: Market Opportunities and the Effective Breadth of a Portfolio. *The Journal of Portfolio Management*, **35**(2), 12-24.

Jacobs, B.I. & Levy, K.N. 2007. Enhanced Active Equity Portfolios are Trim Equitized Long-Short Portfolios. *Journal of Portfolio Management*, **33**(4), 19 -25.

Jobson, J. D., & Korkie, R. M. 1981. Putting Markowitz theory to work. *The Journal of Portfolio Management*, **7**(4), 70–74.

Jorion, P. 1992. Portfolio Optimization in Practice. *Financial Analysts Journal*, **48**(1), 68-74.

Khandani, A. E. & Lo, A.W. 2007. *What Happened to the Quants in August 2007?* Working Paper [online] available: <http://ssrn.com/Summary=1015987>.

Kritzman, M.P. 2003. *The Portable Financial Analyst: What Practitioners Need To Know*, (2nd ed.). Hoboken, New Jersey: John Wiley and Sons, Inc.

Kruger, R. & Van Rensburg, P. 2008. Evaluating and constructing equity benchmarks in the South African portfolio management context, *Investment Analysts Journal*, 67: 5 -18.

Lo, A.W. & Patel, P.N. 2007. 130/30: The New Long-Only, *Journal of Portfolio Management*, **34**(2), 12-38.

Markowitz, H M. 1952. Portfolio Selection. *The Journal of Finance*, **7**, 77-91.

Markowitz, H. M. 1959. *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley & Sons.

Markowitz, H. M. 1999. The Early History of Portfolio Theory: 1600 - 1960. *Financial Analysts Journal*, (July/August), 5-16.

Martielli, J.D. 2005. Quantifying the Benefits of Relaxing the Long-Only Constraint, *SEI Investments*, January.



Michaud, R.O. 1989. The Markowitz Optimization Enigma: Is “Optimized” Optimal? *Financial Analysts Journal*, **45**(1), 31-42.

Polakow, D., Seymour, A. & Greyling, Y. 2007. The South African Alpha Allure: Reflections on the Myths, the Myopia and the Mystique, *Peregrine Securities: Derivative and Quantitative Research*. October.

Polakow, D. 2010. The Long and Active Existentialist. *Journal of Asset Management*.

Ratner, M; Meric, I & Meric, G. 2006. Sector Dispersion and Stock market Predictability. *Journal of Investing*, **15**(1), 56-61.

Raubenheimer, H. 2011. Constraints on Investment Weights: What Mandate Authors in Concentrated Equity Markets Such as South Africa Need to Know. *Investment Analysts Journal*, 74 (forthcoming).

Raubenheimer, H. 2011. Varying Cross-Sectional Volatility in the South African Equity Market and the Implications for the Management of Fund Managers. *South African Journal of Business Management*, **42**(2), 15-26.

Raubenheimer, H. 2010. Concentration in the South African Equity Market and its Implied Restrictions on the Long Only Equity Fund Managers Opportunity Set. *South African Journal of Business Management*, **41**(4).

Raubenheimer, H. & Conradie, H. 2008. Be careful what you wish for – Active Risk is market dependent, *FinWeek - Collective Insights*, Autumn.

Sapra, S.G. 2008. *The Role of Cross-Sectional Dispersion in Active Portfolio Management*. Working paper, [online] available: [ssrn.com/Summary=1266225](http://ssrn.com/Summary=1266225), September.

Scherer, B. & Xu, X. 2007. The Impact of Constraints on Value-Added. *Journal of Portfolio Management*, **33**(4), 45-54.

Sharpe, W.F. 1964. Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *Journal of Finance*, **19**(3), 425-442.

Sharpe, W.F. 1966. Mutual Fund Performance. *Journal of Business*, **39**(1), 119-138.

Sharpe, W.F. 1991. The Arithmetic of Active Management. *Financial Analysts Journal*, **47**(1), January/February, 7-9.

Solnik, B. and Roulet, J. 2000. Dispersion as Cross-Sectional Correlation. *Financial Analysts Journal*, **56**(1), 54-61.

Sorensen, E.H.; Hua, R. & Qian, E. 2007. Aspects of Constrained Long-Short Equity Portfolios. *The Journal of Portfolio Management*, **33**(2), 12-20.

Strongin, S.; Petsch, M. & Sharenow, G. 2000. Beating Benchmarks. *Journal of Portfolio Management*, **26**(4), 11-27.

Stubbs, R.A. & Vendenbussche, D. 2010. Constraint Attribution. *The Journal of Portfolio Management*, 48-59.

Surz, R.J. 1994. Portfolio Opportunity Distributions: An Innovation in Performance Evaluation. *Journal of Investing*, **3**(2), Summer, 36-41.

Surz, R.J. 2007. Accurate Benchmarking is Gone but Not Forgotten: the Imperative Need to get Back to Basics. *The Journal of Performance Measurement*, Spring, 34-43.

Tobin, J. 1958. Liquidity Preference as Behavior Towards Risk. *The Review of Economic Studies*, **67**, February, 65-86.

Van Heerden, J.D., Saunderson, S. 2008. The Effect of the South African Market Concentration on Portfolio Performance. *Corporate Ownership and Control*, **6**(1), 99-106.

Verachia, A. 2011. Global Environment of Business + Drivers and Impact of the BRIC Economies in Africa. Presentation delivered at the CFA South Africa - GIBS Conference, Pretoria, August.

White, H. 1980. A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity. *Econometrica*, **48**(4), 817-838.

Ye, J. 2008. How Variation in Signal Quality Affects Performance. *Financial Analysts Journal*, **64**(4), 48-61.

Yu, W. & Sharaiha, Y. M. 2007. Alpha budgeting – Cross-sectional dispersion decomposed. *Journal of Asset Management*, **8**(1), 58-72.

Zhou, G. 2008. On the Fundamental Law of Active Portfolio Management: What Happens if our Estimates are Wrong? *The Journal of Portfolio Management*, Summer, 26-33.